Study on the 3D anisotropic diffusion and large-scale anisotropy of the galactic cosmic rays

Presenter: Hongbo Hu

Co-author: Wei Liu, Shujie Lin, Yiqing Guo

ICRC2019, Madison, Wisconsin, USA July 28, 2019

Cosmic ray Spectra

Standard Propagation EQ:

Symmetric to GC | Symmetric Source | Diffusion | Convection

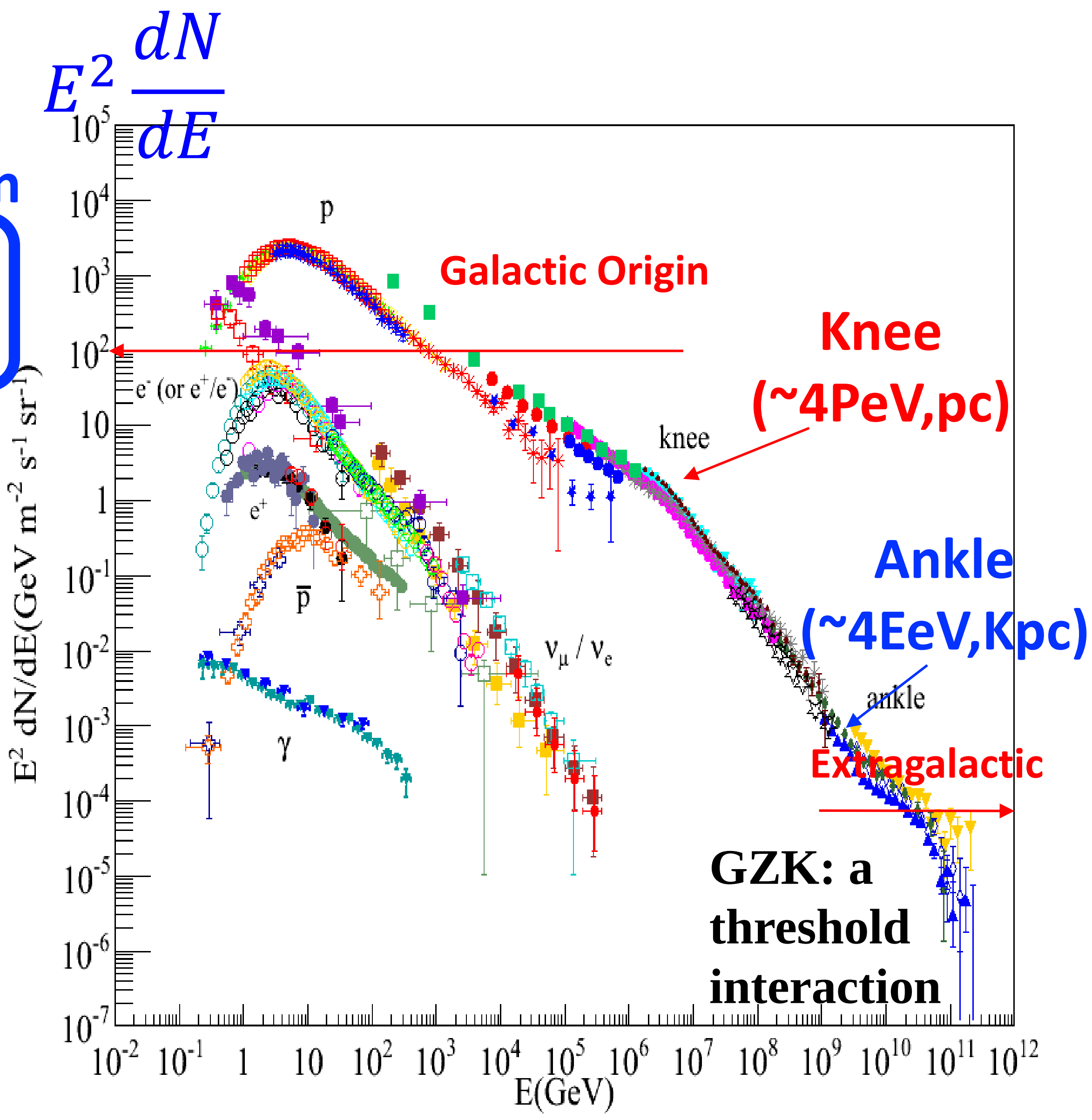
$$\frac{\partial \boxed{\psi(\vec{r},p,t)}}{\partial t} = \boxed{q(\vec{r},p,t)} + \boxed{\sum_i D_{ii}\frac{\partial^2\psi}{\partial x_i^2}} - \boxed{\nabla\cdot(\vec{V}_c\psi)}$$

$$+ \frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial}{\partial p}\frac{1}{p^2}\psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla\cdot V_c\psi)\right]$$

$$\boxed{-\frac{\psi}{\tau_f} - \frac{\psi}{\tau_r}}$$

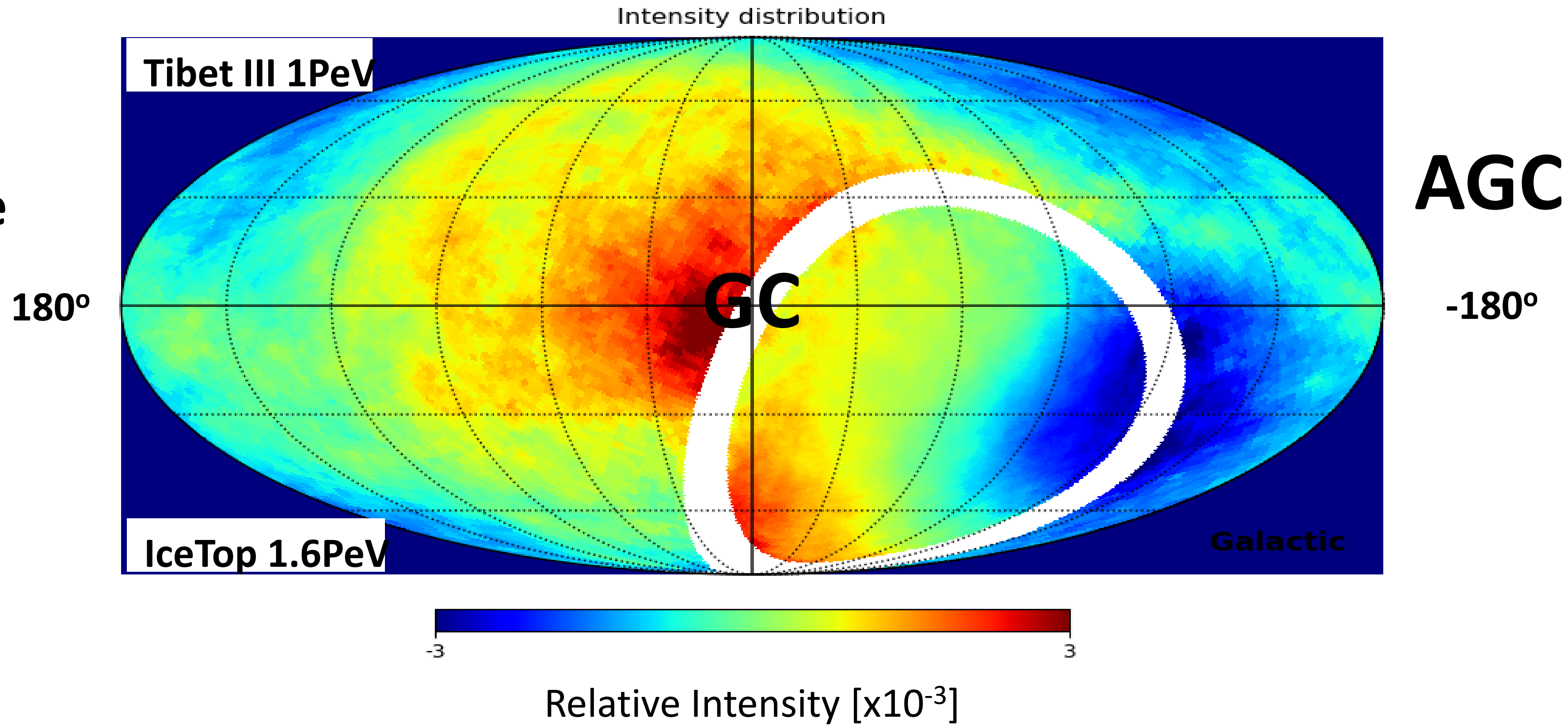
Secondary production

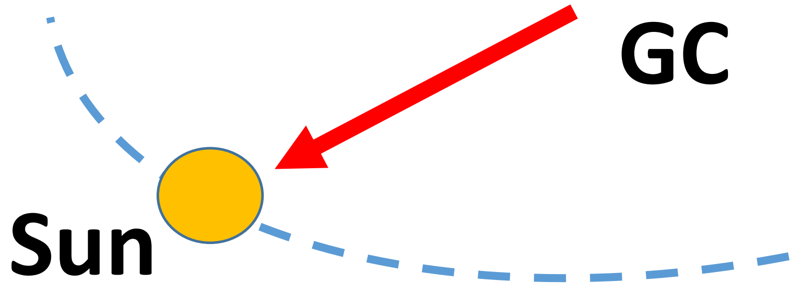
- Spectra of CRs, ratio between secondary and primary CRs, eg. B/C are well described;
- Makes prediction, eg., anisotropy;



PeV CRs anisotropy in Galactic coordinate

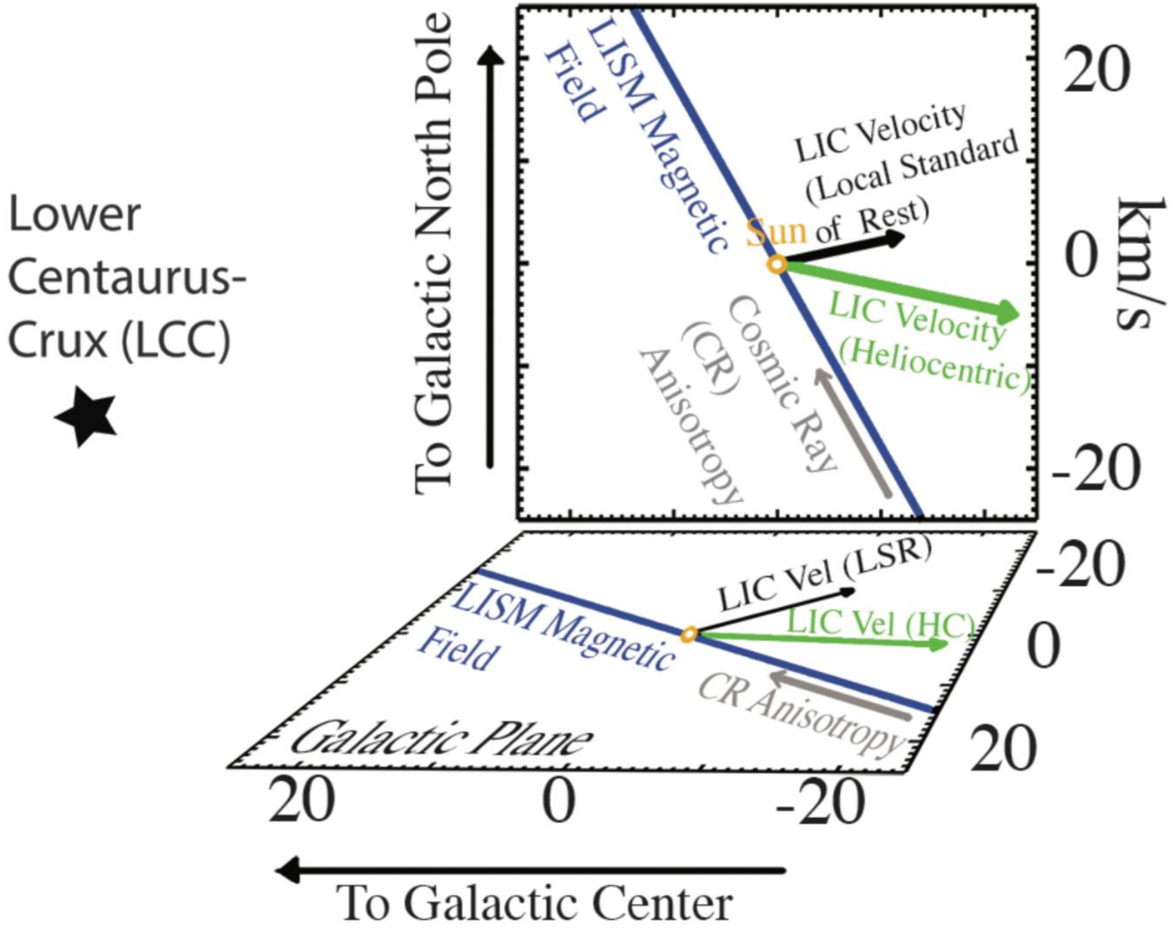
- **Standard model predicts that the anisotropy of GCRs are of dipole form, with the direction pointing to the GC .**
- **Agree with the observation above 100TeV.**



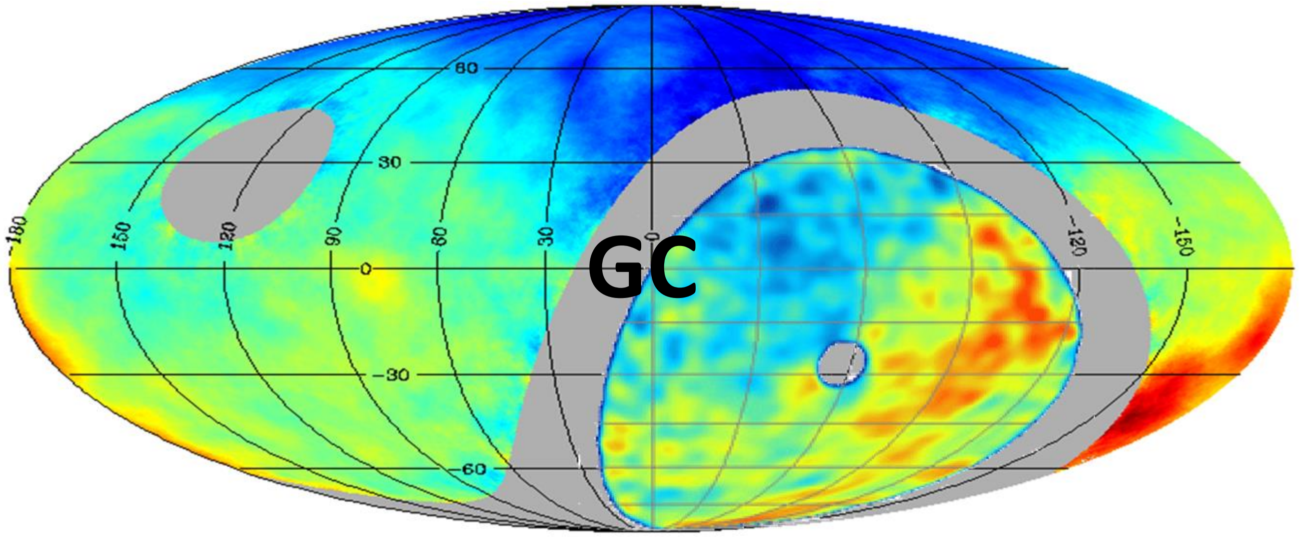


Consistent with the GLOBAL diffusion prediction;
Services as an indication that PeV CRs are galactic origin.

TeV CRs anisotropy in the Galactic coordinates



N. A. Schwadron, et al., 2014, Science



1. **Below 100 TeV, Dipole does not point to GC, but to the Local ISMF direction**
2. **Indicates that regular magnetic field Is important to the propagation of GCRs**

Anisotropic diffusion in regular magnetic field

Under ordered magnetic field $\vec{B}$, diffusion process is anisotropic and D_{ij} is rank-two symmetric tensor

In a field-aligned coordinate system

$$D_{ij} = \begin{pmatrix} D_{\|} & 0 & 0 \\ 0 & D_{\perp} & 0 \\ 0 & 0 & D_{\perp} \end{pmatrix}$$

$D_{\|}$ **diffusion coefficient aligned with $\vec{B}$**

$$D_{\|} = D_{0\|} \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\|}} \qquad \mathcal{R}_0 = 4\mathrm{GV}$$

$D_{\perp}$ **diffusion coefficient perpendicular to $\vec{B}$**

$$D_{\perp} = D_{0\perp} \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\perp}} \equiv \varepsilon D_{0\|} \left(\frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\perp}} \qquad \varepsilon = \frac{D_{0\perp}}{D_{0\|}}$$

Off-diagonal terms in the general coordinate system

$$D_{ij} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

$$D_{ij} \equiv D_\perp \delta_{ij} + \left(D_\parallel - D_\perp\right) b_i b_j \qquad b_i = \frac{B_i}{|B|}$$

J. Giacalone and J. R. Jokipii, 1999, ApJ

diffusion term

$$\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) + \boxed{\sum_{ij} \frac{\partial}{\partial x_i}\left(D_{ij} \frac{\partial \psi}{\partial x_j}\right)} - \nabla \cdot (\vec{V}_c \psi)$$

$$+ \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla \cdot V_c \psi)\right] - \frac{\psi}{\tau_f} - \frac{\psi}{\tau_r}$$

Pseudo source solution to off-diagonal diffusion terms

$$
\begin{aligned}
& q(\vec{r}, p, t) + \nabla \cdot (D \nabla \psi) \\
& = \left[q(\vec{r}, p, t) + \sum_{i \neq j} \frac{\partial}{\partial x_i} \left(D_{ij} \frac{\partial \psi}{\partial x_j} \right) \right] + \sum_i \frac{\partial}{\partial x_i} \left(D_{ii} \frac{\partial \psi}{\partial x_i} \right) \\
& = [q(\vec{r}, p, t) + q_{\mathrm{pseudo}}(\vec{r}, p, t)] + \sum_i \frac{\partial}{\partial x_i} \left(D_{ii} \frac{\partial \psi}{\partial x_i} \right)
\end{aligned}
$$

q_{pseudo} **is called pseudo source term**

A Form of GALPROP Equation

New propagation equation becomes

$$\frac{\partial \psi(\vec{r},p,t)}{\partial t} = q(\vec{r},p,t) + q_{\rm pseudo}(\vec{r},p,t) + \sum_i \frac{\partial}{\partial x_i}\left(D_{ii}\frac{\partial \psi}{\partial x_i}\right)$$

$$- \nabla\cdot(\vec{V}_c\psi) + \frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial}{\partial p}\frac{1}{p^2}\psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla\cdot V_c\psi)\right]$$

$$- \frac{\psi}{\tau_f} - \frac{\psi}{\tau_r} \ .$$

Iteration method

(1) assume $q_{\rm pseudo}^{(0)} = 0$, solve propagation equation to obtain distribution $\psi^{(1)}$

(2) calculate $q_{\rm pseudo}^{(1)}$ according to $\psi^{(1)}$, then solve propagation equation with $q_{\rm pseudo}^{(1)}$ to obtain the new distribution $\psi^{(2)}$

repeat procedure (2) for n times until $|\psi^{(n+1)} - \psi^{(n)}|$ become small enough

Test by a toy magnetic field model

DRAGON solved a 2D anisotropic propagation problem

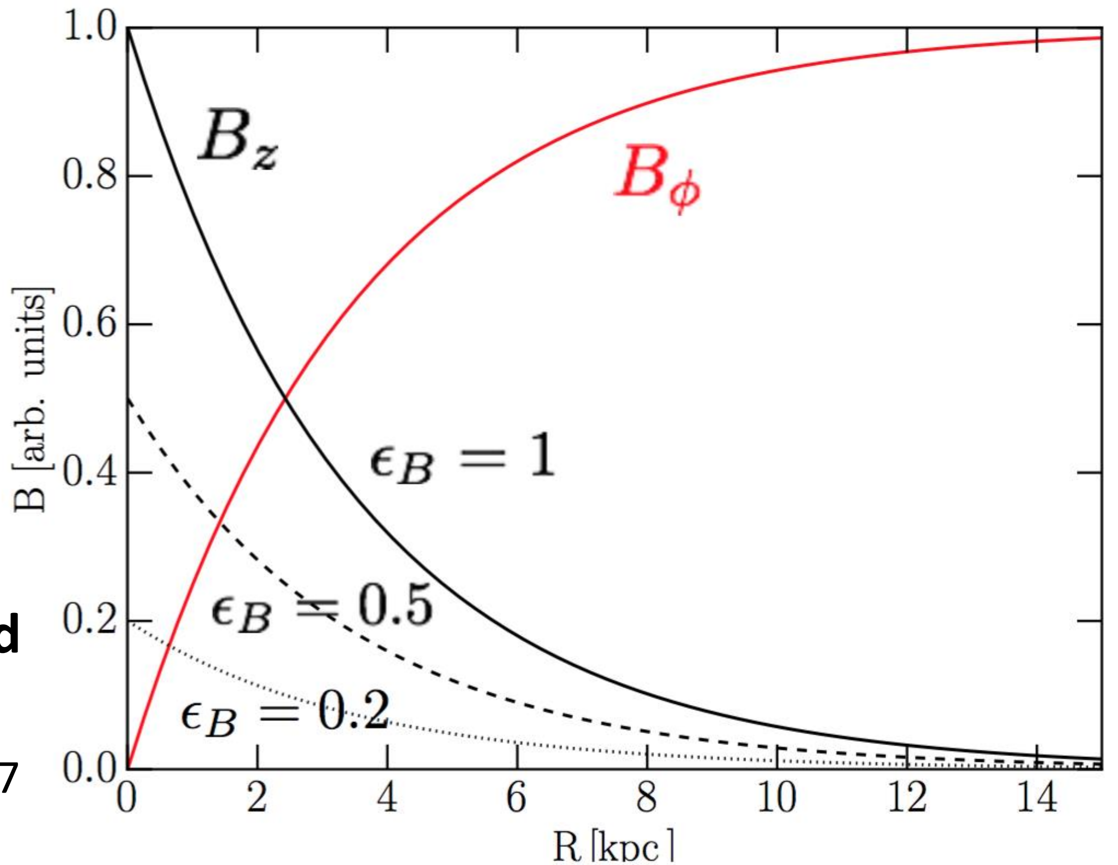
C. Evoli, D. Gaggero, et al. 2017, JCAP

$$B_r = 0$$

$$B_\phi = B_{0,\phi}\left(1 - \exp\left[-\frac{r}{r_0}\right]\right)$$

$$B_z = B_{0,z}\exp\left[-\frac{r}{r_0}\right] \equiv \varepsilon_B B_{0,\phi}\exp\left[-\frac{r}{r_0}\right]$$

B_ϕ is dominated far away from the Galactic center



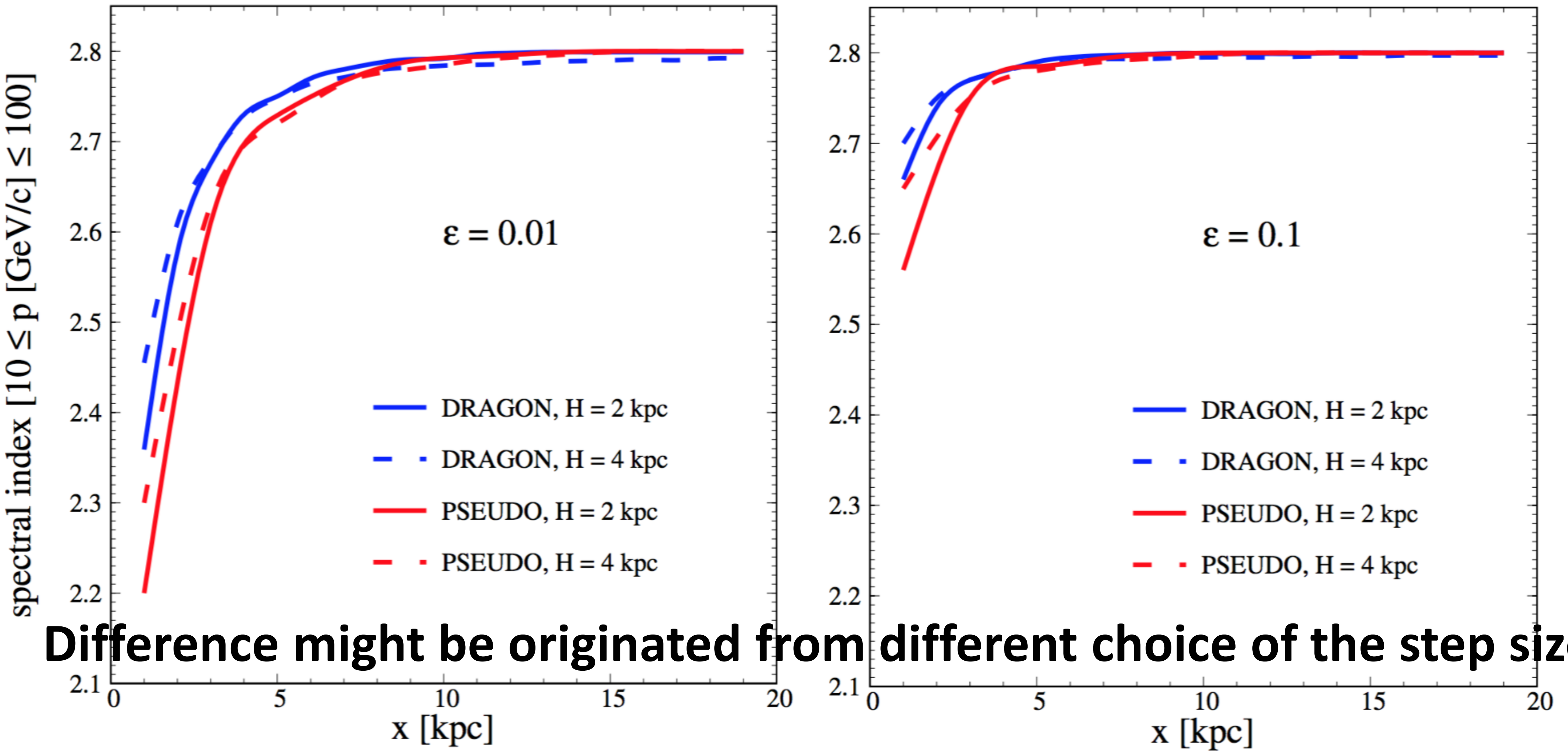
Close to the Galactic center, B_z is dominated

from Andrea Vittino's talk @ICRC2017

$D_{\parallel}$ dominated when $r \ll R_0$

$D_{\perp}$ dominated when $r \gg R_0$

$$\psi(\mathcal{R}) = \begin{cases} \mathcal{R}^{-(\alpha+\delta_{\parallel})}\,, & r \ll R_0 \\ \mathcal{R}^{-(\alpha+\delta_{\perp})}\,, & r \gg R_0 \end{cases}$$



$y = 0, z = 0$

$\varepsilon_B = 0.2$

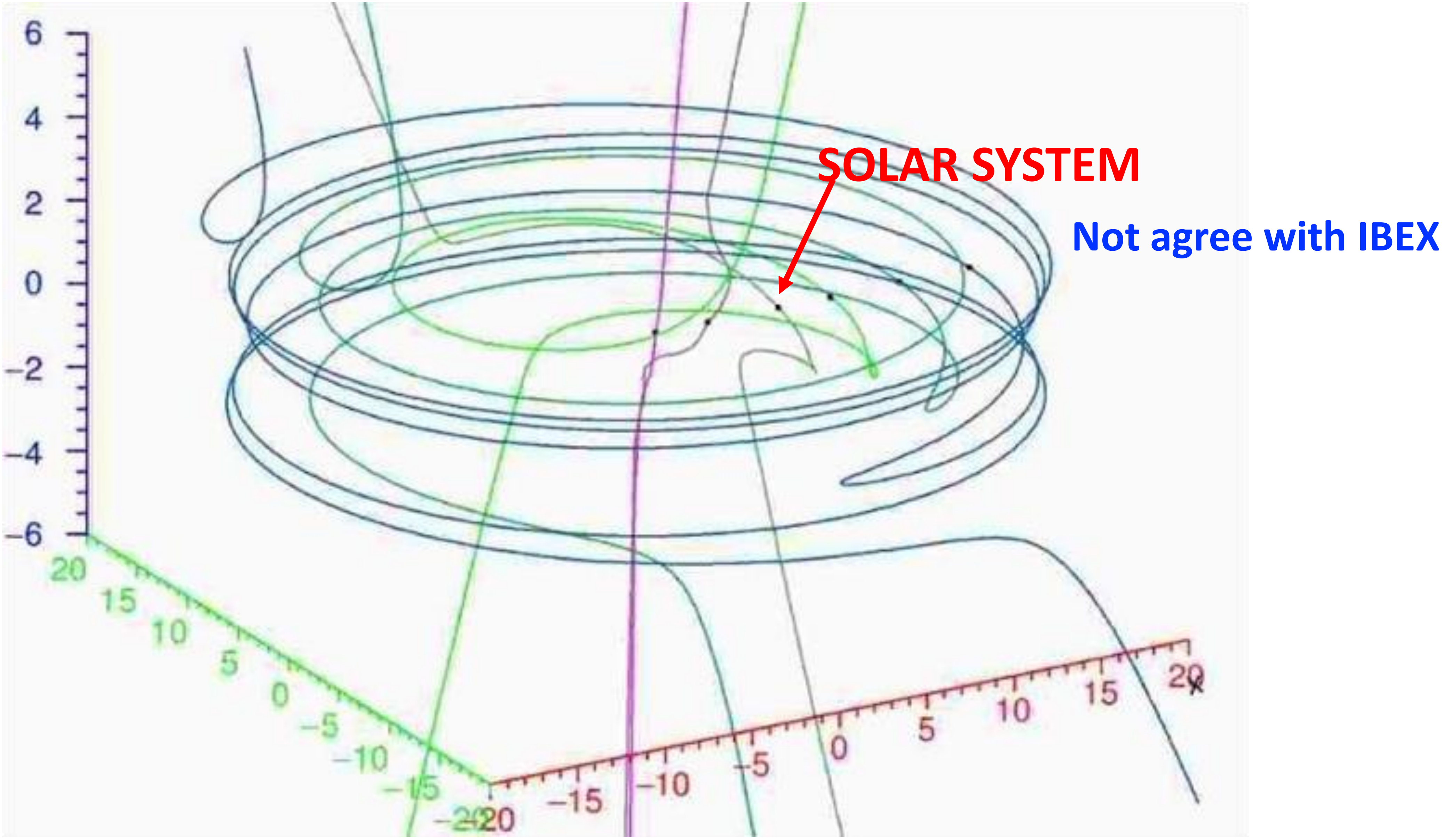
$\delta_{\parallel} = 0.1$

$\delta_{\perp} = 0.6$

$R_0 = 3.5$ kpc

Difference might be originated from different choice of the step size of grid

A Realistic 3D Galactic Magnetic Field Model



Formulae of the Model

disk component

$$B_\phi^{\rm disk}(r,z) = \begin{cases} B_0^{\rm d} \exp\left(-\frac{|z|}{z_0}\right) \quad , & r \leqslant R_c^{\rm d} \\ B_0^{\rm d} \exp\left(-\frac{(r-r_\odot)}{R_0} - \frac{|z|}{z_0}\right) \quad , & r > R_c^{\rm d} \end{cases}$$

poloidal component

$$B_r^{\rm pol}(r,z) = B_{\rm X}(r,z) \exp\left(-\frac{R^{\rm p}}{R^{\rm X}}\right) \sin\Theta_{\rm X} \, ,$$

$$B_z^{\rm pol}(r,z) = B_{\rm X}(r,z) \exp\left(-\frac{R^{\rm p}}{R^{\rm X}}\right) \cos\Theta_{\rm X} \, ,$$

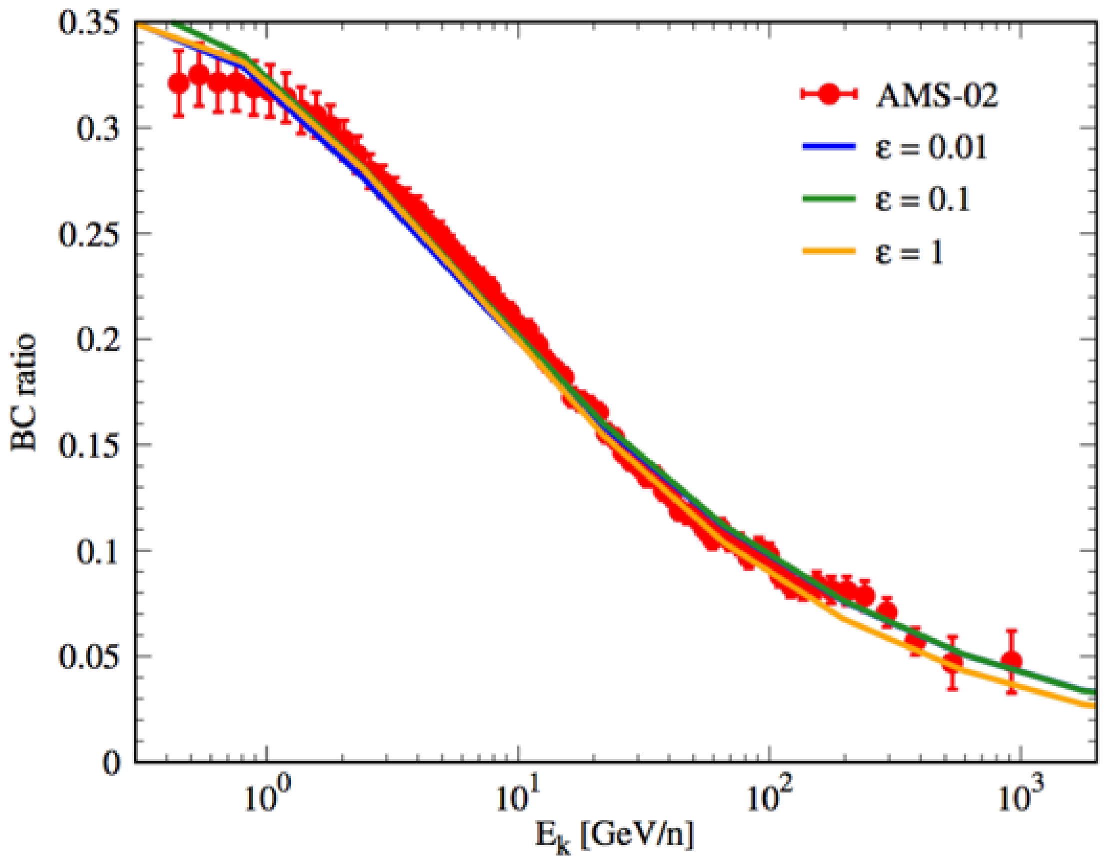
R. Jansson and G. R. Farrar, 2012, ApJ

halo component

$$B_\phi^{\rm halo}(r,z) = B_0^{\rm h} \left[1 + \left(\frac{|z| - z_0^{\rm h}}{z_1^{\rm h}}\right)\right]^{-1} \times \frac{r}{R_0^{\rm h}} \exp\left(1 - \frac{r}{R_0^{\rm h}}\right)$$

M. S. Pshirkov, P. G. Tinyakov et al, 2011, ApJ

Result of B/C ratio



Nearby the solar system, perpendicular diffusion is dominated

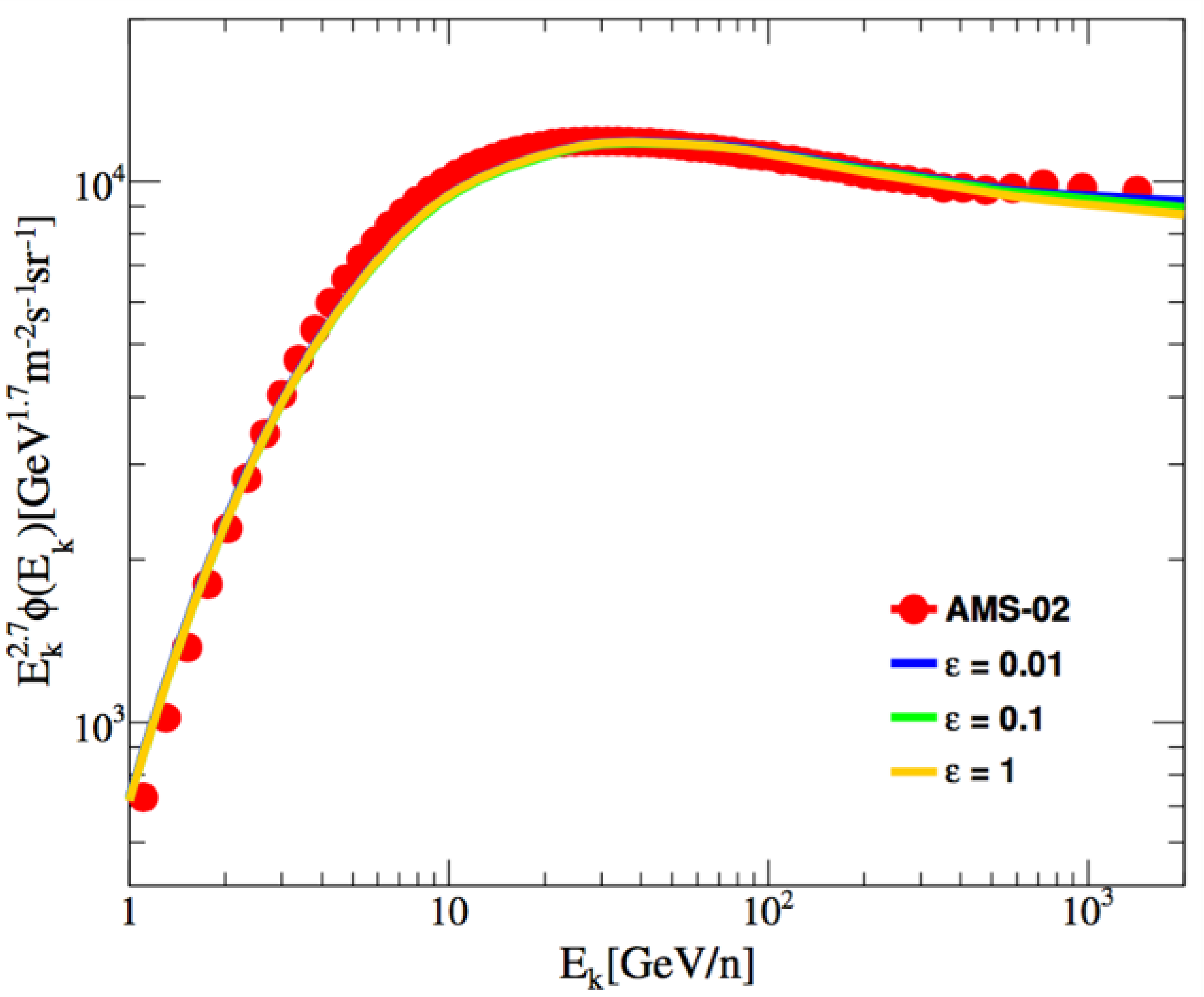
$\delta_\perp$ is determined by the B/C ratio(L is fixed to 4 kpc)

$\delta_\parallel$ is fixed to 0.3

When ε drops, both $D_{0\parallel}$ accordingly increases

	ε	$D_{0\parallel}$ [cm²/s]	$\delta_\parallel$	$\delta_\perp$	η	ν_1	$\mathcal{R}_0$ [GV]	ν_2
Set 1	1.0	5.6×10^{28}	0.3	0.5	0.5	2.44	350	2.34
Set 2	0.1	3.6×10^{29}	0.3	0.5	0.5	2.44	350	2.34
Set 3	0.01	1.3×10^{30}	0.3	0.48	0.5	2.44	350	2.34

Result of proton spectrum



Summary

A pseudo source method is proposed to solve the 3D anisotropic diffusion equation with existing GALPROP package.

B/C ratio and proton spectrum under a realistic magnetic field model are reproduced with reasonable parameters.

CR Anisotropy will be studied with magnetic field model capable representing local ISMF in the future.

Thank you for your attention