Study on the 3D anisotropic diffusion and large-scale anisotropy of the galactic cosmic rays

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**Cosmic ray Spectra**

Standard Propagation EQ:

\[
\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) + \sum_i D_{ii} \frac{\partial^2 \psi}{\partial x_i^2} - \nabla \cdot (\vec{V}_c \psi)
\]

\[
+ \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi - \frac{\partial}{\partial p} \left[ \dot{\psi} - \frac{p}{3} (\nabla \cdot V_c \psi) \right]
\]

- Secondary production

- Spectra of CRs, ratio between secondary and primary CRs, eg. B/C are well described;

- Makes prediction, eg., anisotropy;

**Symmetric to GC**

**Symmetric**

**Source**

**Diffusion**

**Convection**

Galactic Origin

Knee (~4PeV,pc)

Ankle (~4EeV,Kpc)

GZK: a threshold interaction

Secondary production
PeV CRs anisotropy in Galactic coordinate

- Standard model predicts that the anisotropy of GCRs are of dipole form, with the direction pointing to the GC.
- Agree with the observation above 100TeV.

Consistent with the GLOBAL diffusion prediction; Services as an indication that PeV CRs are galactic origin.
TeV CRs anisotropy in the Galactic coordinates

1. Below 100 TeV, Dipole does not point to GC, but to the Local ISMF direction
2. Indicates that regular magnetic field is important to the propagation of GCRs

N. A. Schwadron, et al., 2014, Science
Anisotropic diffusion in regular magnetic field

Under ordered magnetic field $\vec{B}$, diffusion process is anisotropic and $D_{ij}$ is rank-two symmetric tensor.

In a field-aligned coordinate system

$$D_{ij} = \begin{pmatrix} D_{\parallel} & 0 & 0 \\ 0 & D_{\perp} & 0 \\ 0 & 0 & D_{\perp} \end{pmatrix}$$

$D_{\parallel}$ diffusion coefficient aligned with $\vec{B}$

$$D_{\parallel} = D_{0\parallel} \left( \frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\parallel}} \quad \mathcal{R}_0 = 4\text{GV}$$

$D_{\perp}$ diffusion coefficient perpendicular to $\vec{B}$

$$D_{\perp} = D_{0\perp} \left( \frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\perp}} \equiv \varepsilon D_{0\parallel} \left( \frac{\mathcal{R}}{\mathcal{R}_0} \right)^{\delta_{\parallel}} \quad \varepsilon = \frac{D_{0\perp}}{D_{0\parallel}}$$
Off-diagonal terms in the general coordinate system

\[ D_{ij} = \begin{pmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{yx} & D_{yy} & D_{yz} \\
D_{zx} & D_{zy} & D_{zz}
\end{pmatrix} \]

\[ D_{ij} \equiv D_\perp \delta_{ij} + (D_\| - D_\perp) b_i b_j \quad \quad b_i = \frac{B_i}{|B|} \]


\[
\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) + \sum_{ij} \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial \psi}{\partial x_j} \right) - \nabla \cdot (\vec{V}_c \psi)
\]

\[
+ \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi - \frac{\partial}{\partial p} \left[ \dot{p} \psi - \frac{p}{3} (\nabla \cdot V_c \psi) \right] - \frac{\psi}{\tau_f} - \frac{\psi}{\tau_r}
\]
Pseudo source solution to off-diagonal diffusion terms

\[ q(\vec{r}, p, t) + \nabla \cdot (D \nabla \psi) \]

\[ = \left[ q(\vec{r}, p, t) + \sum_{i \neq j} \frac{\partial}{\partial x_i} \left( D_{ij} \frac{\partial \psi}{\partial x_j} \right) \right] + \sum_{i} \frac{\partial}{\partial x_i} \left( D_{ii} \frac{\partial \psi}{\partial x_i} \right) \]

\[ = [q(\vec{r}, p, t) + q_{\text{pseudo}}(\vec{r}, p, t)] + \sum_{i} \frac{\partial}{\partial x_i} \left( D_{ii} \frac{\partial \psi}{\partial x_i} \right) \]

\( q_{\text{pseudo}} \) is called pseudo source term
A Form of GALPROP Equation

New propagation equation becomes

\[
\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = q(\vec{r}, p, t) + q_{\text{pseudo}}(\vec{r}, p, t) + \sum_i \frac{\partial}{\partial x_i} \left( D_{ii} \frac{\partial \psi}{\partial x_i} \right) \\
- \nabla \cdot (\vec{V}_c \psi) + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi - \frac{\partial}{\partial p} \left[ \dot{\psi} - \frac{p}{3} (\nabla \cdot V_c \psi) \right] \\
- \frac{\psi}{\tau_f} - \frac{\psi}{\tau_r}.
\]
Iteration method

(1) assume $q_{\text{pseudo}}^{(0)} = 0$, solve propagation equation to obtain distribution $\psi^{(1)}$

(2) calculate $q_{\text{pseudo}}^{(1)}$ according to $\psi^{(1)}$, then solve propagation equation with $q_{\text{pseudo}}^{(1)}$ to obtain the new distribution $\psi^{(2)}$

repeat procedure (2) for $n$ times until $|\psi^{(n+1)} - \psi^{(n)}|$ become small enough
Test by a toy magnetic field model

DRAGON solved a 2D anisotropic propagation problem

\[ B_r = 0 \]
\[ B_\phi = B_{0,\phi} \left(1 - \exp\left[-\frac{r}{r_0}\right]\right) \]
\[ B_z = B_{0,z} \exp\left[-\frac{r}{r_0}\right] \equiv \epsilon_B B_{0,\phi} \exp\left[-\frac{r}{r_0}\right] \]

\( B_\phi \) is dominated far away from the Galactic center

Close to the Galactic center, \( B_z \) is dominated

\( \epsilon_B \) is varied

from Andrea Vittino’s talk @ICRC2017
\( D_{\parallel} \text{ dominated when } r \ll R_0 \)

\( D_{\perp} \text{ dominated when } r \gg R_0 \)

\[
\psi(R) = \begin{cases} 
\mathcal{R}^{-(\alpha + \delta_{\parallel})}, & r \ll R_0 \\
\mathcal{R}^{-(\alpha + \delta_{\perp})}, & r \gg R_0
\end{cases}
\]

\[ y = 0, z = 0 \]

\[ \varepsilon_B = 0.2 \]

\[ \delta_{\parallel} = 0.1 \]

\[ \delta_{\perp} = 0.6 \]

\[ R_0 = 3.5 \text{ kpc} \]

Difference might be originated from different choice of the step size of grid.
A Realistic 3D Galactic Magnetic Field Model

Not agree with IBEX
Formulae of the Model

**disk component**

\[
B^\text{disk}_\phi (r, z) = \begin{cases} 
B^d_0 \exp \left( - \frac{|z|}{z_0} \right), & r \leq R^d_c \\
B^d_0 \exp \left( - \frac{(r - r_\odot)}{R_0} - \frac{|z|}{z_0} \right), & r > R^d_c
\end{cases}
\]

**poloidal component**

\[
B^\text{pol}_r (r, z) = B_X(r, z) \exp \left( - \frac{R^p}{R^X} \right) \sin \Theta_X,
\]

\[
B^\text{pol}_z (r, z) = B_X(r, z) \exp \left( - \frac{R^p}{R^X} \right) \cos \Theta_X,
\]


**halo component**

\[
B^\text{halo}_\phi (r, z) = B^h_0 \left[ 1 + \left( \frac{|z| - z^h_0}{z^h_1} \right) \right]^{-1} \times \frac{r}{R^h_0} \exp \left( 1 - \frac{r}{R^h_0} \right)
\]

Result of B/C ratio

Nearby the solar system, perpendicular diffusion is dominated

\( \delta_\perp \) is determined by the B/C ratio (\( L \) is fixed to 4 kpc)

\( \delta_\parallel \) is fixed to 0.3

When \( \varepsilon \) drops, both \( D_{0\parallel} \) accordingly increases

<table>
<thead>
<tr>
<th>Set</th>
<th>( \varepsilon )</th>
<th>( D_{0\parallel} ) [cm(^2)/s]</th>
<th>( \delta_\parallel )</th>
<th>( \delta_\perp )</th>
<th>( \eta )</th>
<th>( \nu_1 )</th>
<th>( R_0 ) [GV]</th>
<th>( \nu_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>1.0</td>
<td>( 5.6 \times 10^{28} )</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>2.44</td>
<td>350</td>
<td>2.34</td>
</tr>
<tr>
<td>Set 2</td>
<td>0.1</td>
<td>( 3.6 \times 10^{29} )</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>2.44</td>
<td>350</td>
<td>2.34</td>
</tr>
<tr>
<td>Set 3</td>
<td>0.01</td>
<td>( 1.3 \times 10^{30} )</td>
<td>0.3</td>
<td>0.48</td>
<td>0.5</td>
<td>2.44</td>
<td>350</td>
<td>2.34</td>
</tr>
</tbody>
</table>
Result of proton spectrum
Summary

A pseudo source method is proposed to solve the 3D anisotropic diffusion equation with existing GALPROP package.

B/C ratio and proton spectrum under a realistic magnetic field model are reproduced with reasonable parameters.

CR Anisotropy will be studied with magnetic field model capable representing local ISMF in the future.
Thank you for your attention