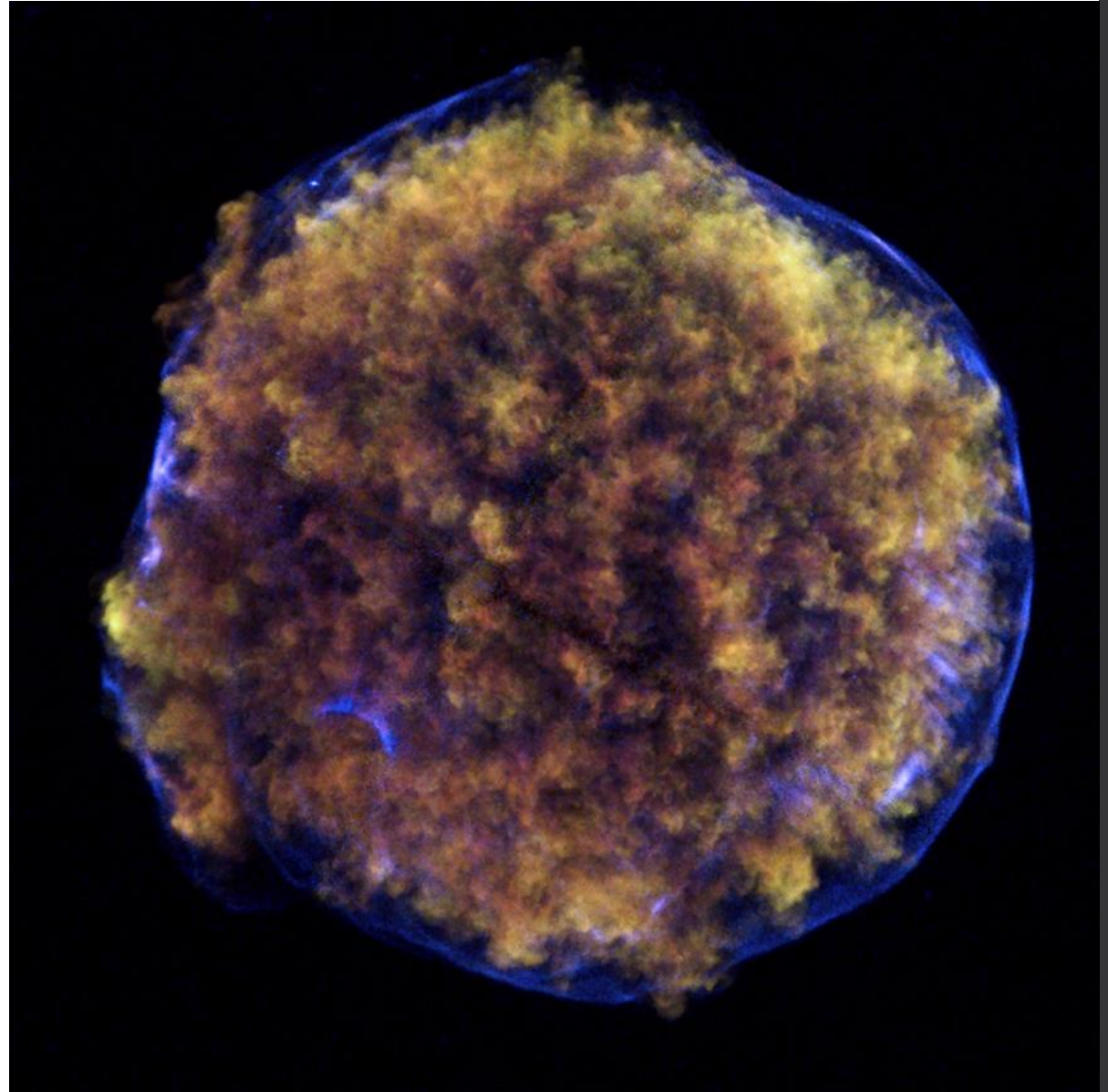# Modeling the Saturation of the Bell Instability using Hybrid Simulations

Georgios Zacharegkas, Damiano Caprioli and Colby Haggerty

ICRC 2019

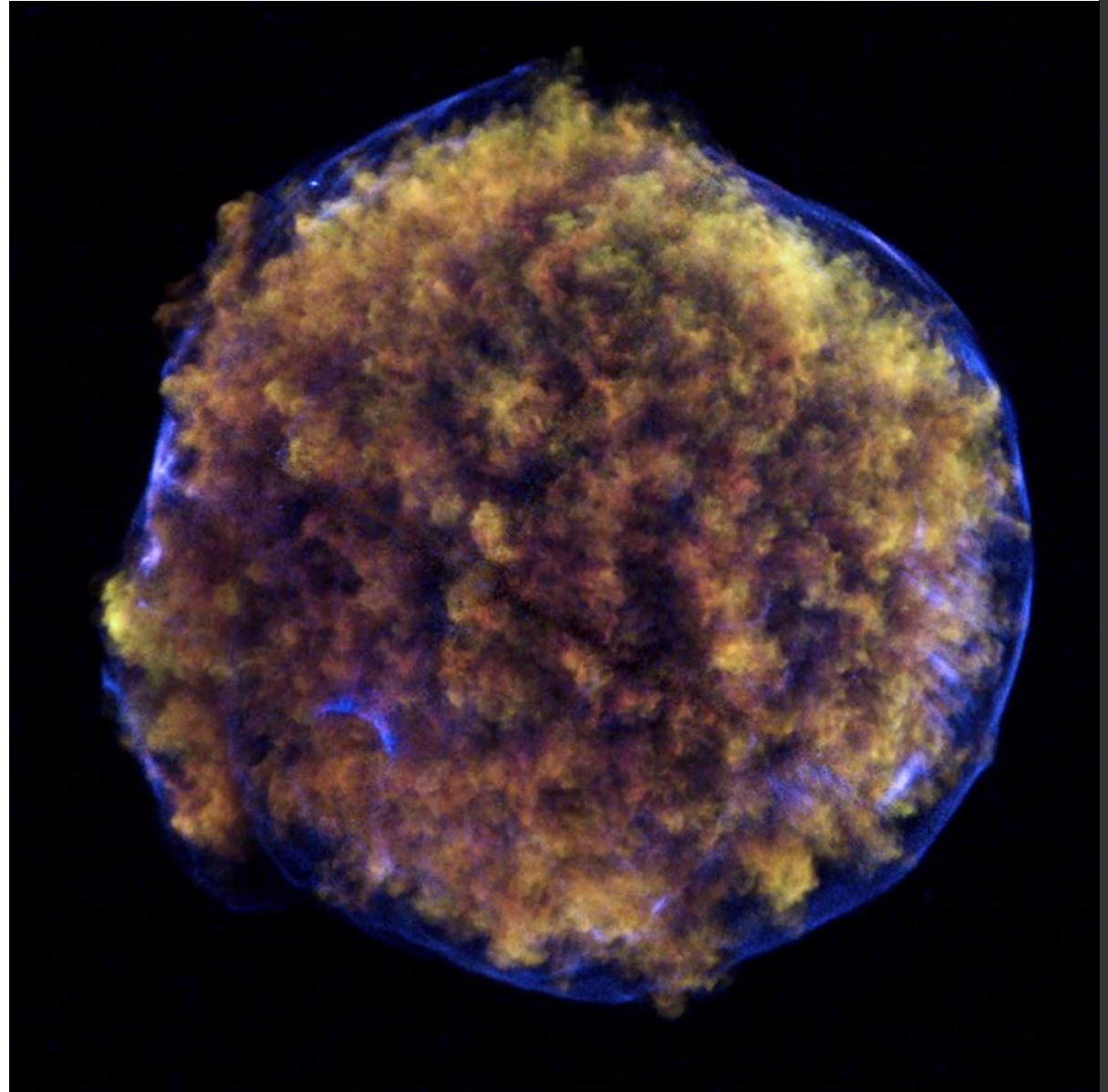# Magnetic field amplification at SNR

Plasma instabilities

Magnetic field amplification at SNR

Plasma instabilities

I) Resonant (e.g., Kulsrud & Pearce1969; McKenzie & Volk1982)

Saturation at $\delta B/B_0 \approx 1$

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Magnetic field amplification at SNR
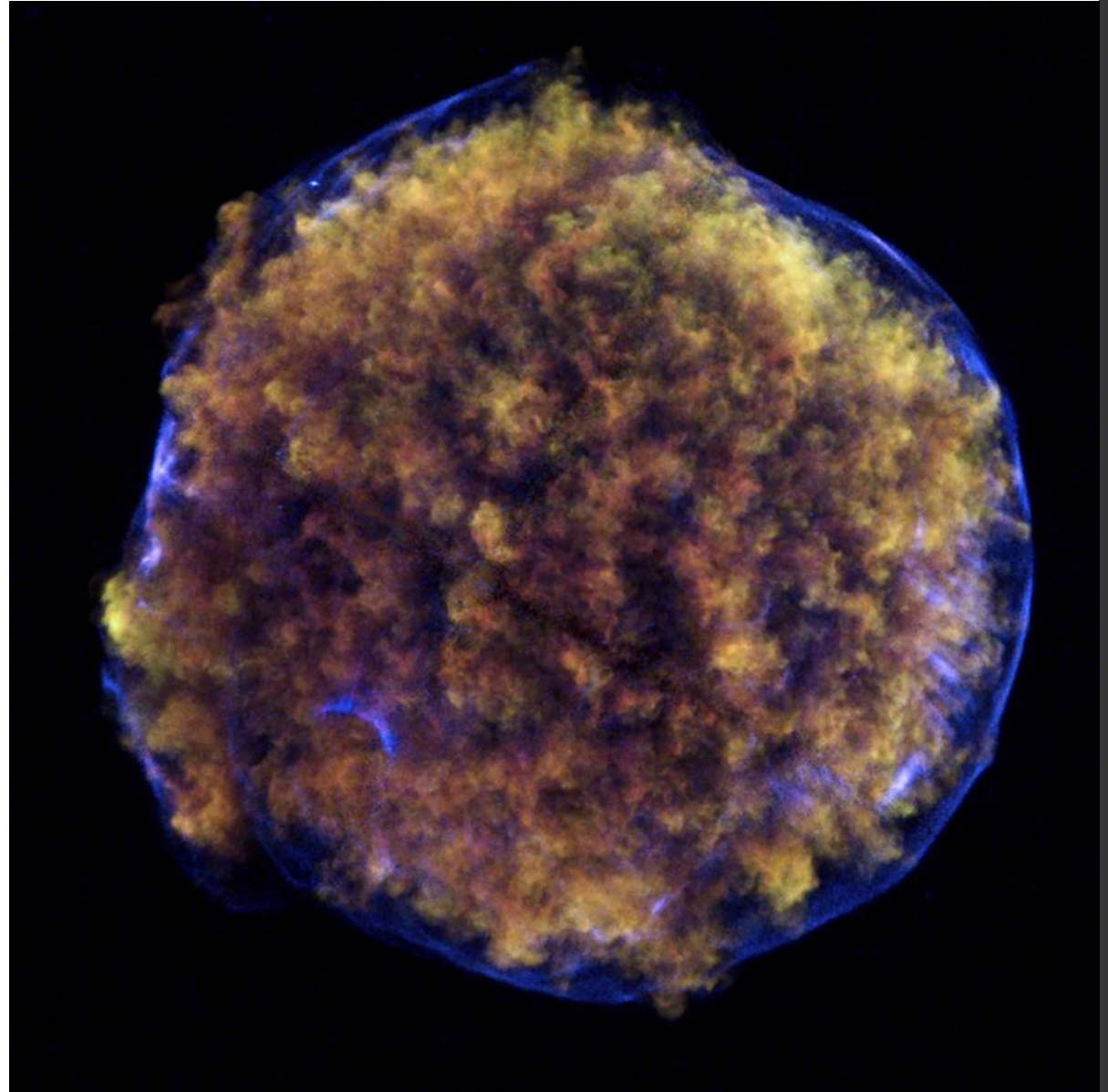
Plasma instabilities

I) Resonant (e.g., Kulsrud & Pearce1969; McKenzie & Volk1982)

Saturation at $\delta B / B_0 \approx 1$

II) Non-resonant (Bell 2004; Reville et al. 2006; Blasi & Amato 2007; Zirakashvili et al. 2008; Niemiec et al. 2008; Riquelme and Spitkovsky 2009; Zweibel et al 2010)

Large amplification factors: $\delta B / B_0 \gtrsim 10 - 100$

Fast: $\tau_{NR} \sim 1 \text{ year} \ll \tau_{SNR}$
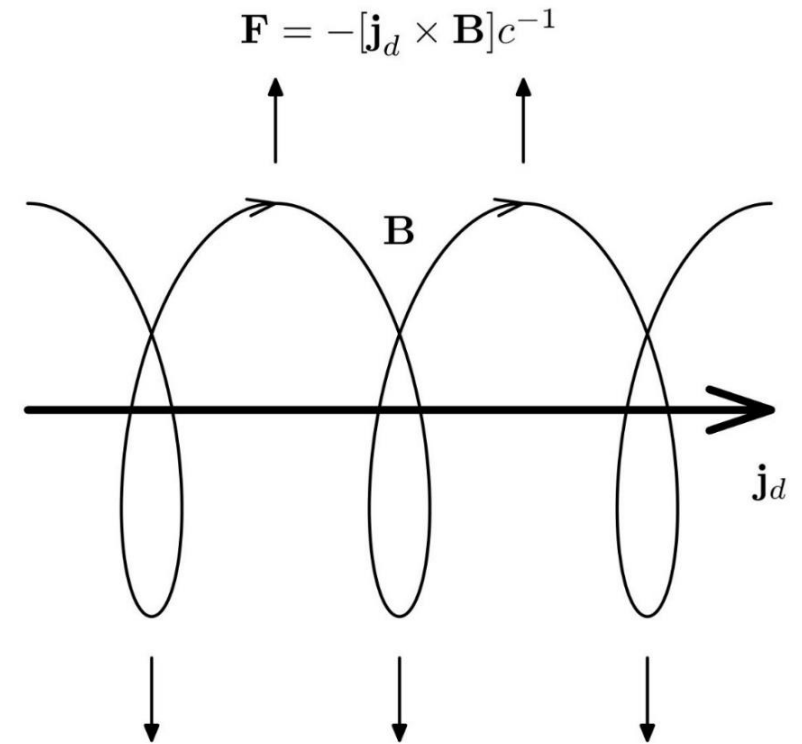
How are the fields generated?

Why do they saturate?

How much do they grow?

The Bell instability

- CRs stream in plasma
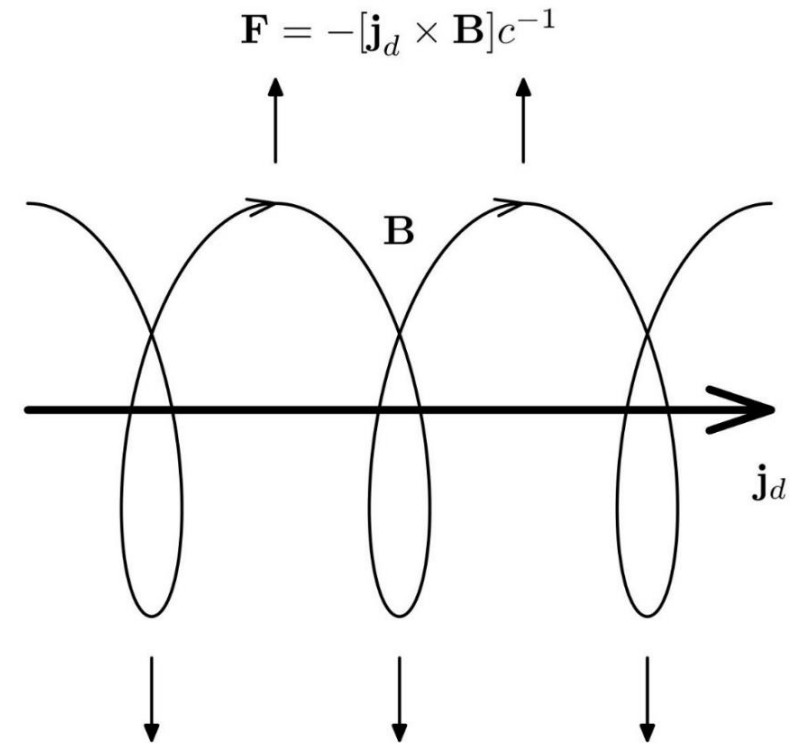
- The return current stretches and amplifies the field

Credit: Zirakashvili, Ptuskin & Volk 2008

$$\mathbf{F} = -[\mathbf{j}_d \times \mathbf{B}]c^{-1}$$

The Bell instability

- CRs stream in plasma

- The return current stretches and amplifies the field

- Fastest growing mode: $k_{\max} = \dfrac{1}{2}\left(\dfrac{n_{CR}}{n_g}\right)\left(\dfrac{v_{CR}}{v_A}\right)d_i^{-1}$

- Growth rate: $\gamma_{\max} = k_{\max}v_A$

Credit: Zirakashvili, Ptuskin & Volk 2008

$$\mathbf{F} = -[\mathbf{j}_d \times \mathbf{B}]c^{-1}$$

$\mathbf{B}$

$\mathbf{j}_d$

Simulations

dHybridR scheme:

- Electrons considered a fluid

- Protons are treated kinetically

(Gargate et al 2007; Haggerty & Caprioli 2019, in prep.)

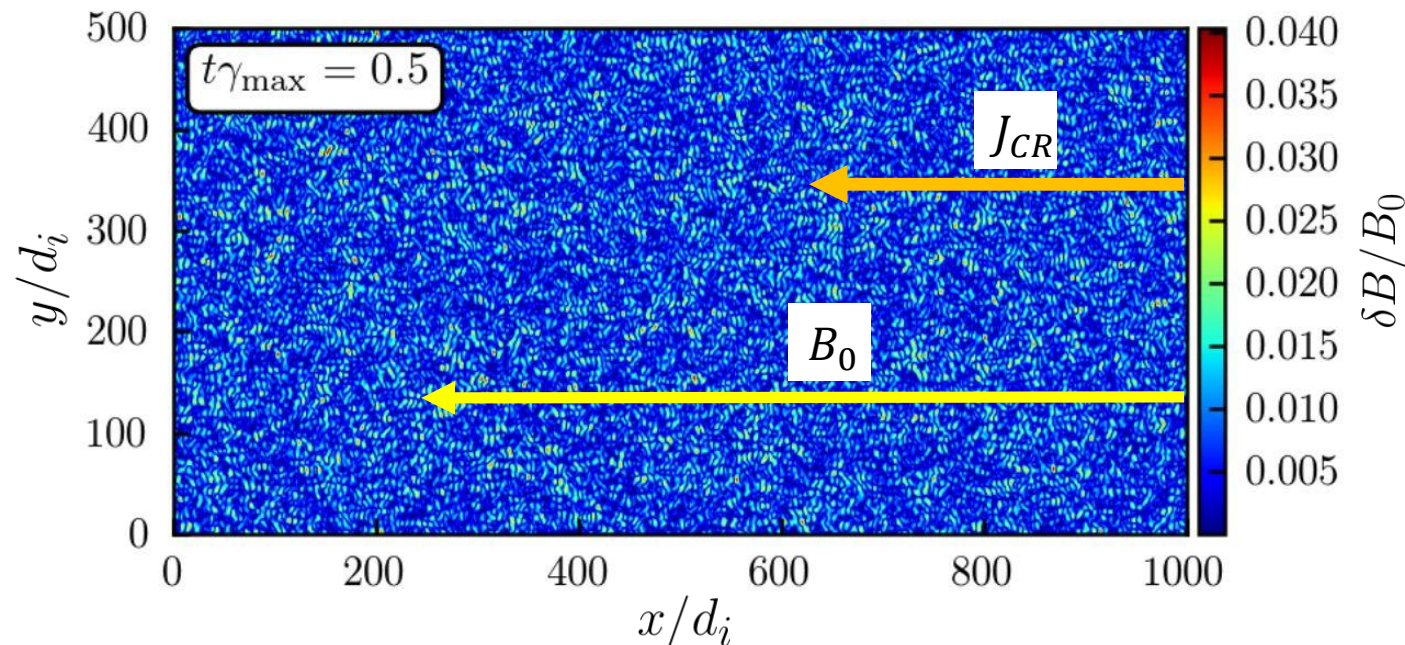**Simulations are 2.5D:**

- 2D in space

- 3D in momentum and fields

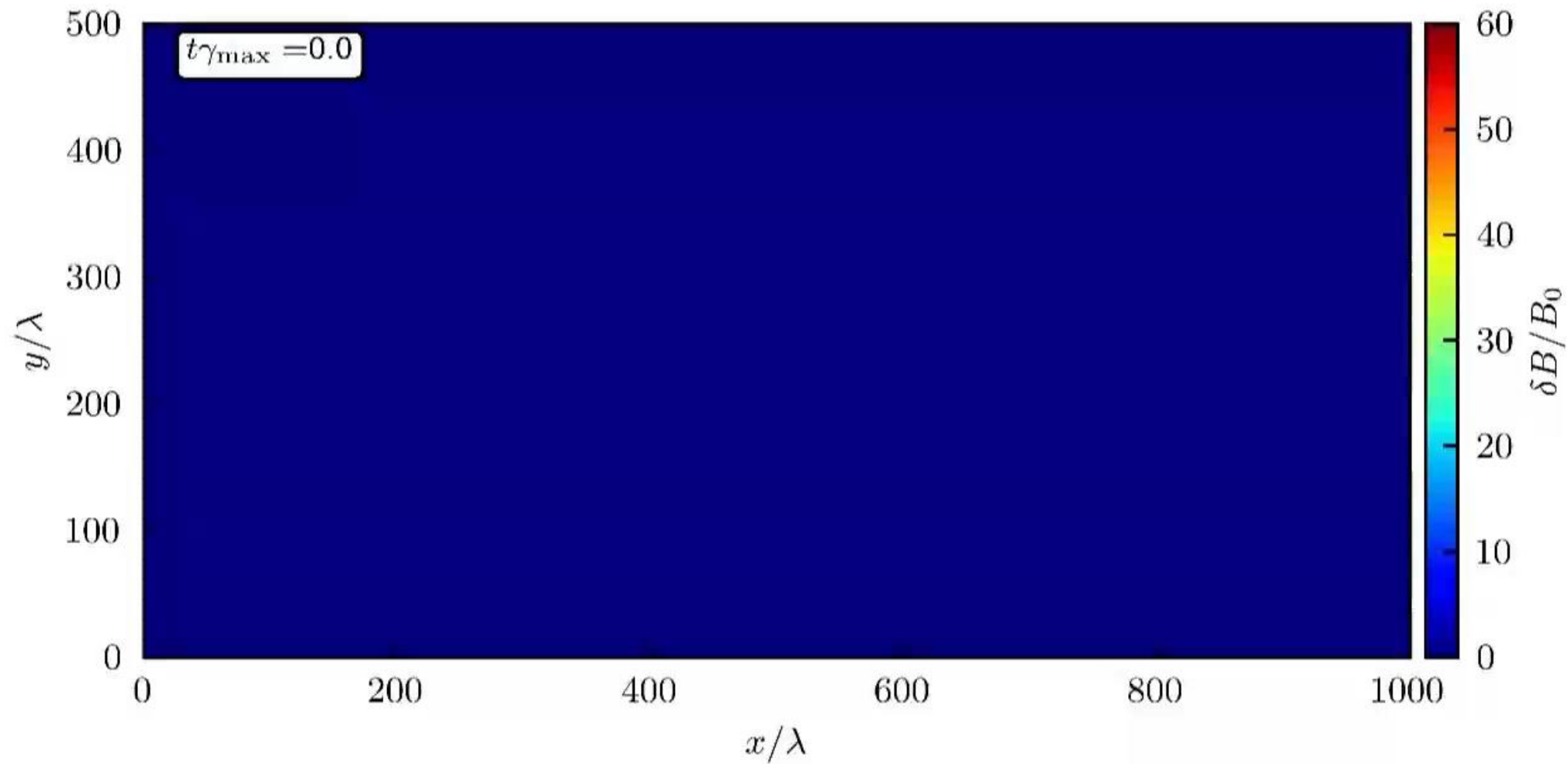**Common parameters:**

$$p_{CR}/m\gamma_{CR} \approx c$$
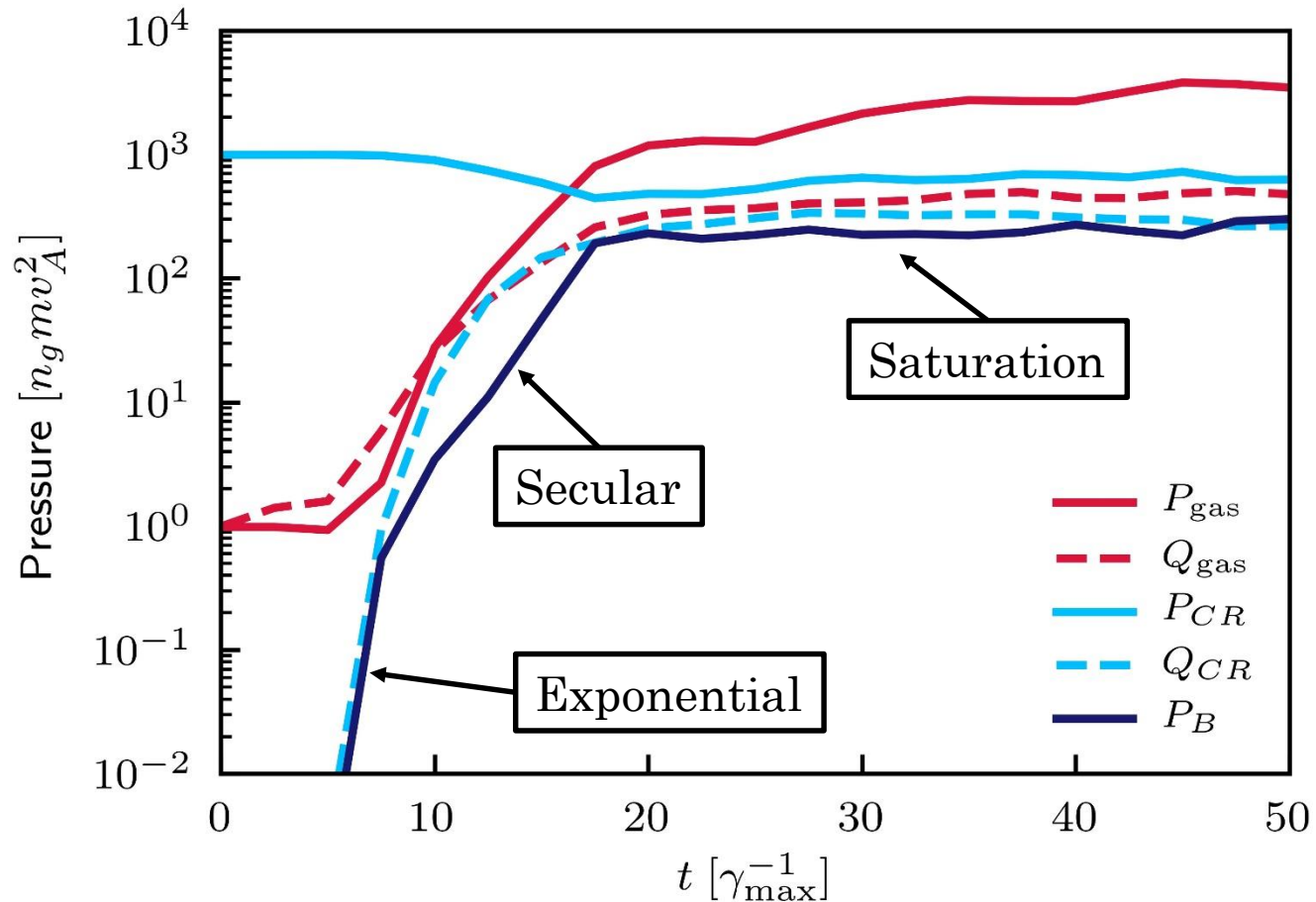$$p_{\mathrm{iso}} \ll p_{CR}$$
$$v_{\mathrm{th}}/v_A = 1$$
$$c/v_A = 100$$

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Turbulence generation

Pressure evolution

Simulation parameters

$n_{CR}/n_g = 10^{-2}$

$p_{CR}/mv_A = 10^3$

$c/v_A = 100$

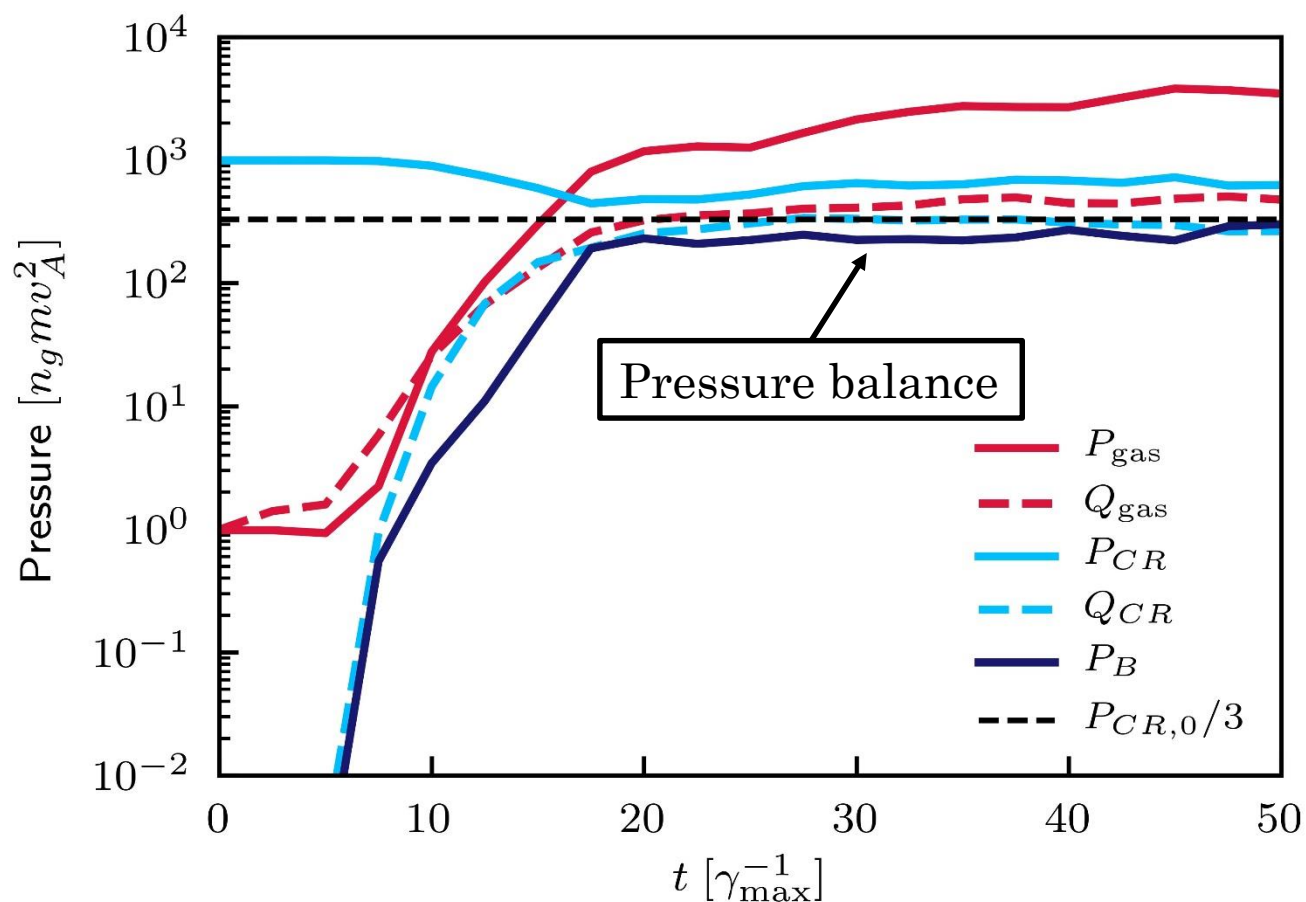# Saturation of the Bell instability

Saturation condition: $Q_{\text{gas}} \approx P_{CR,0}/3$

Simulation parameters

$n_{CR}/n_g = 10^{-2}$

$p_{CR}/mv_A = 10^3$

$c/v_A = 100$

Pressure balance

Legend:
- P_{gas}
- Q_{gas}
- P_{CR}
- Q_{CR}
- P_B
- $P_{CR,0}/3$

Axes: Pressure $[n_g m v_A^2]$ vs $t\ [\gamma_{\max}^{-1}]$

Field amplification – Bell regime

- Magnetic field pressure ~ CR pressure

- Consistent with equipartition between magnetic field and CRs (e.g. Bell 2004; Blasi et al 2015)
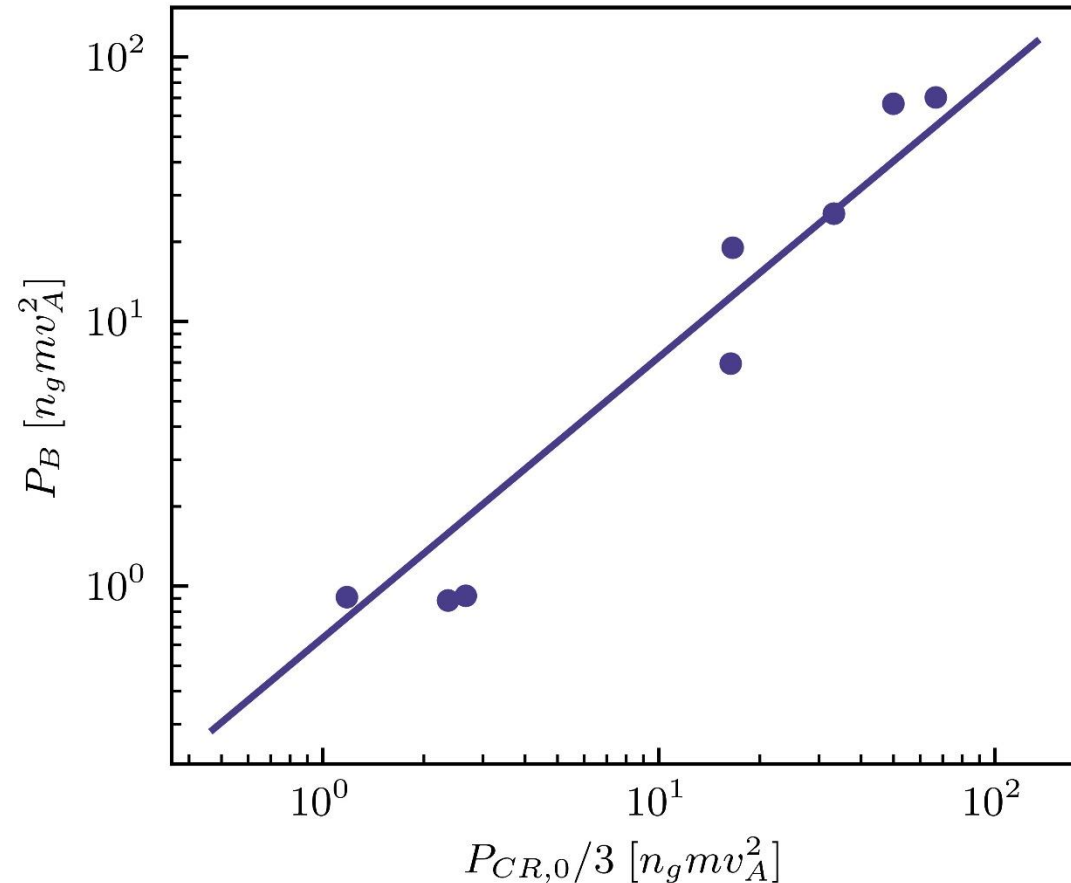
However, this is not the full story!

Thermally modified Bell instability

Effects from the thermal ions
Resonant interactions between ions and waves

"Classical" Bell $\longrightarrow$ "Modified" Bell – WICE
(Zweibel et al 2010)

$$k_{\max} r_{\mathrm{g,th}} \to 0$$

$$k_{\max} r_{\mathrm{g,th}} \sim 1$$

The WICE instability

Condition to be in the "WICE regime"

$$\left(\frac{n_{CR}}{n_g}\right)\left(\frac{v_{CR}}{v_A}\right) > \left(\frac{v_A}{v_{\text{th}}}\right)^2$$

Strong CR currents

Hot plasma

The WICE instability

Condition to be in the "WICE regime"

$$\left(\frac{n_{CR}}{n_g}\right)\left(\frac{v_{CR}}{v_A}\right) > \left(\frac{v_A}{v_{\text{th}}}\right)^2$$

Strong CR currents

Hot plasma

Growth is suppressed

$$\gamma_{WICE} \sim \left(\frac{n_{CR}}{n_g}\frac{v_{CR}}{v_A}\frac{v_A}{v_{\text{th}}}\right)^{2/3}\Omega_{ci} < \gamma_{Bell}$$
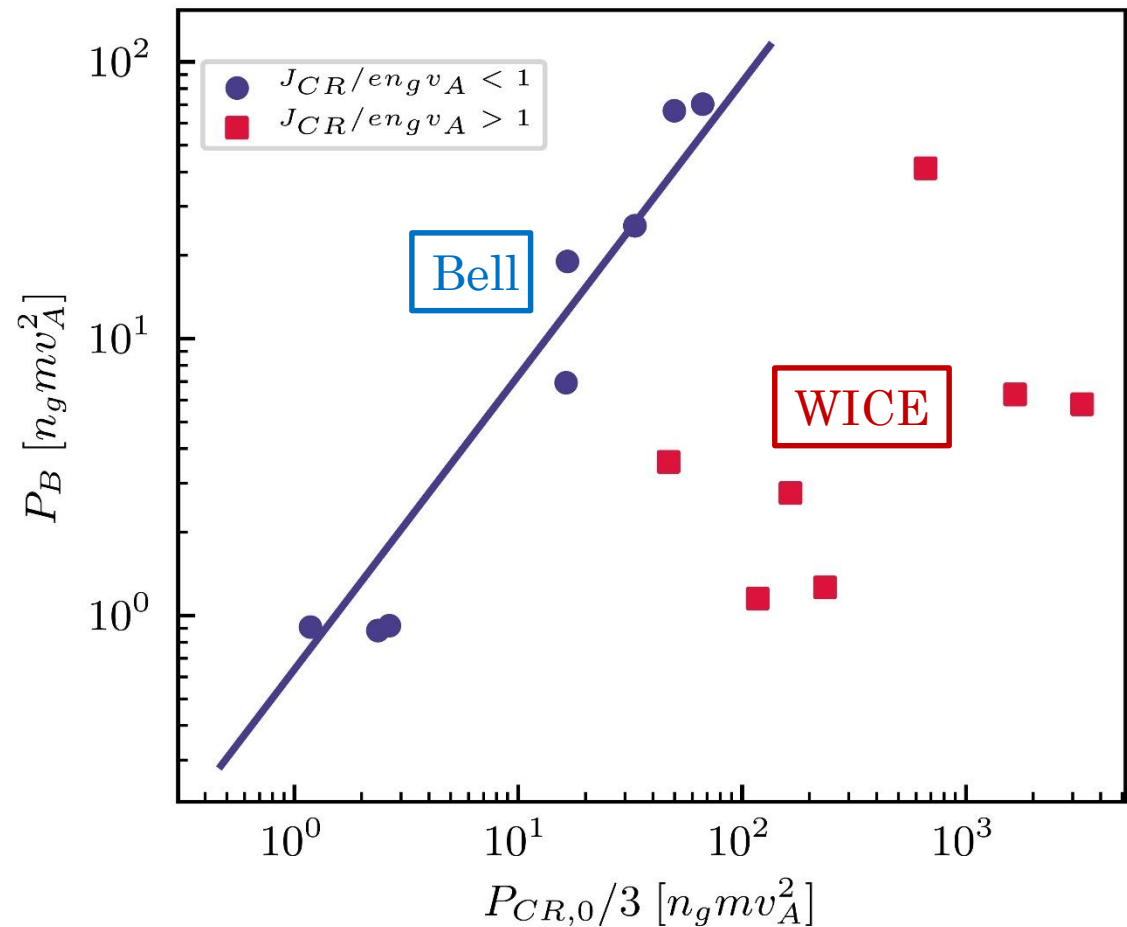
Field amplification – WICE regime

Bell regime
Magnetic field pressure ~ CR pressure

WICE regime
Magnetic field is below equipartition
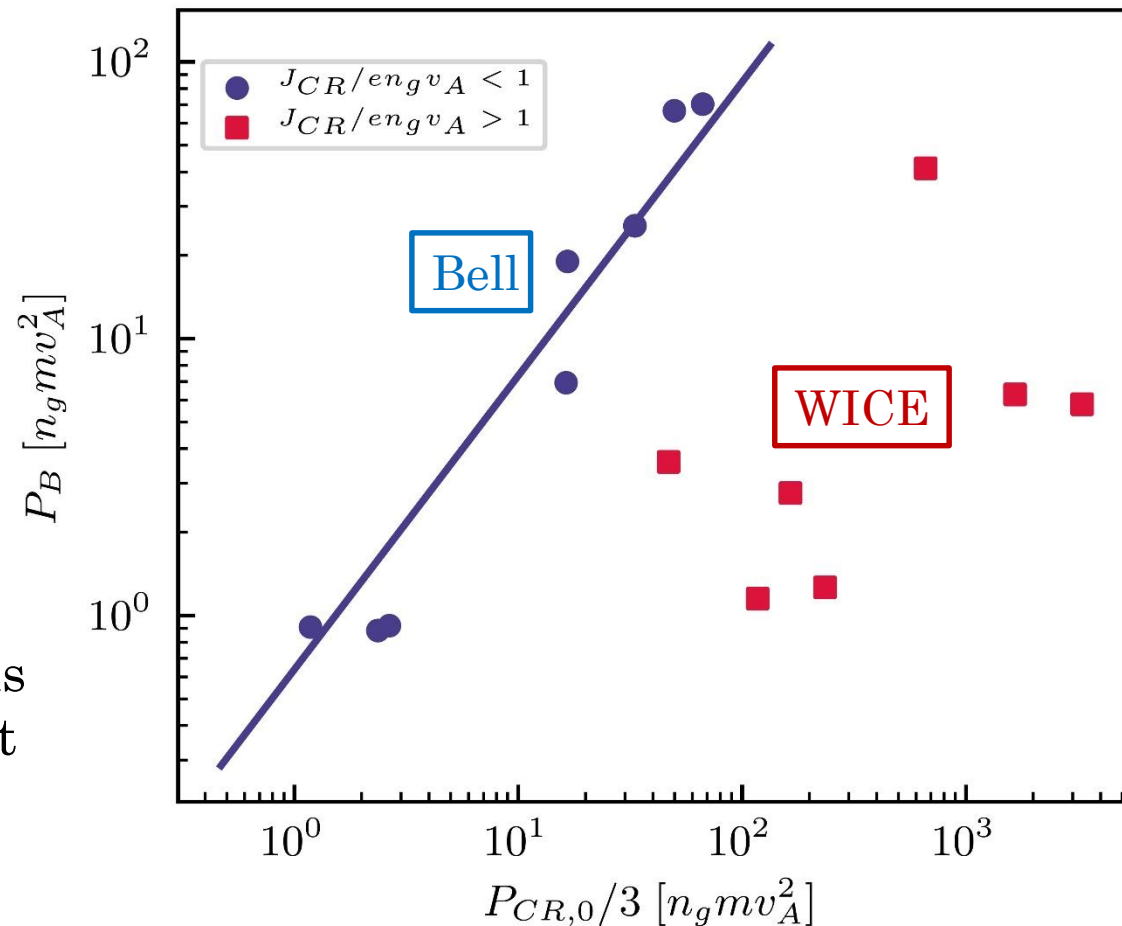
Field amplification – WICE regime

Bell regime

Magnetic field pressure ~ CR pressure
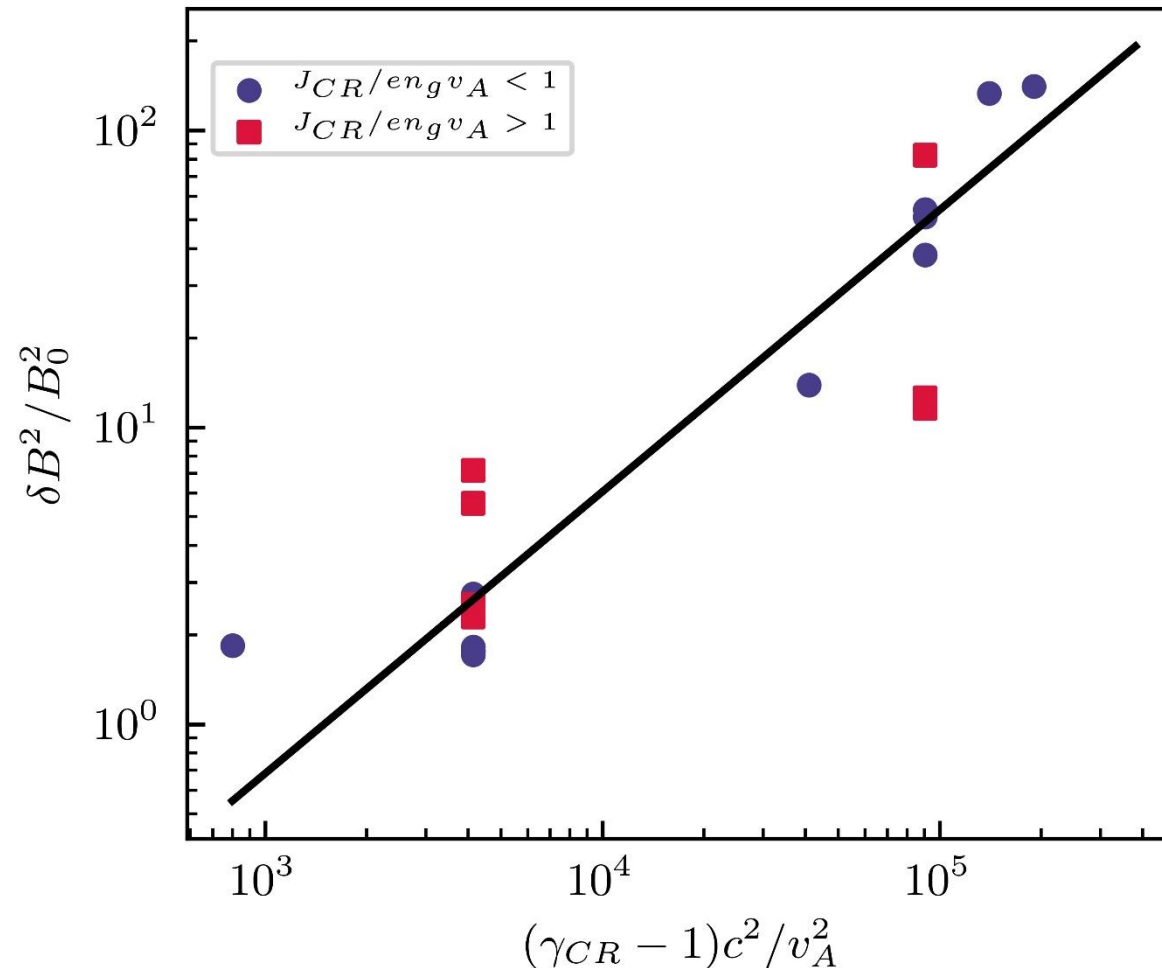
WICE regime

Magnetic field is below equipartition

This might explain the low amplification levels seen in some literature which are inconsistent with the Bell instability (e.g. Niemiec et al. 2008)

Amplification level at saturation

Correlation between final magnetic field and kinetic energy per CR particle

Conclusions

- **Saturation:** Result of pressure balance between gas and CRs, in the perpendicular direction

- **Bell regime:** The system is close to a state of equipartition between gas, CRs and magnetic field pressure

- **WICE regime:** The magnetic pressure is below the gas-CR equipartition

- **Generally**: The magnetic field at saturation is well correlated with the kinetic energy per CR particle

Backup slides

Bell to WICE transition
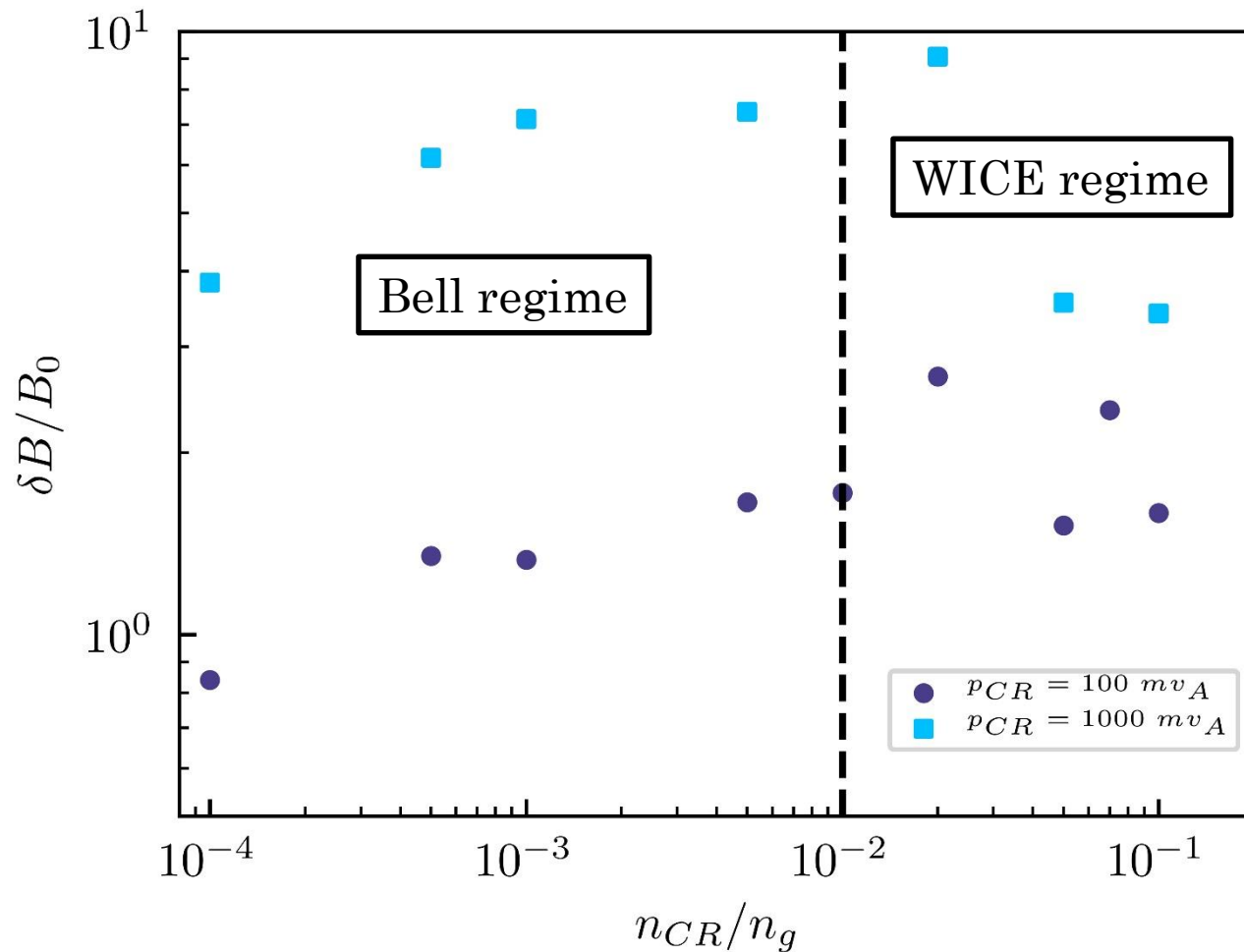
When the CR current becomes strong enough the instability changes behavior
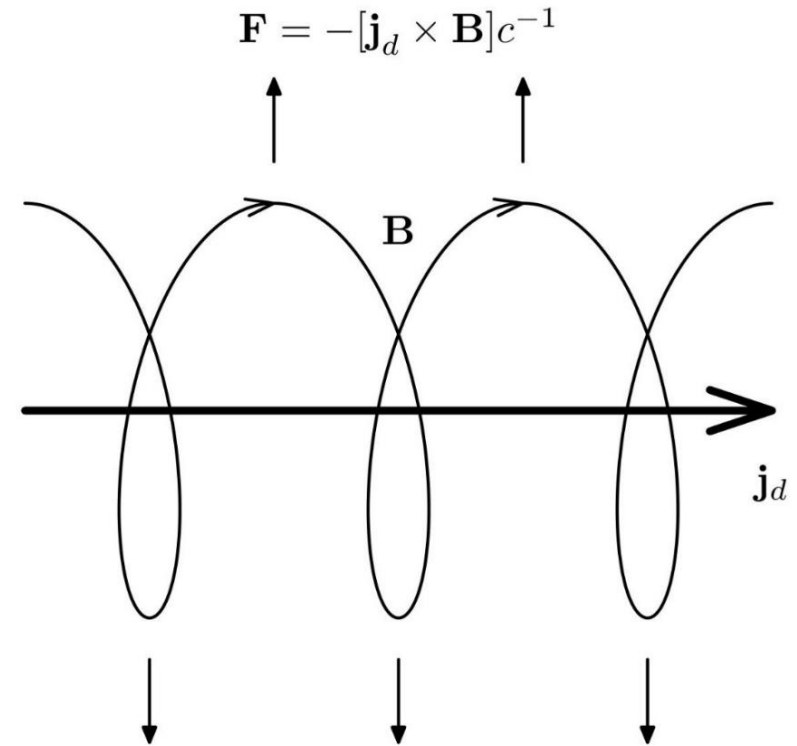
Conditions for Bell

Conditions for the Bell instability to grow

- Driving force must dominate over magnetic tension:

$$\mathbf{J}_{CR} \times \mathbf{B} > \frac{c}{4\pi}\mathbf{B} \times (\nabla \times \mathbf{B})$$

- CRs are non-resonant: $k_{\max} r_{CR} \gg 1$

$$\mathbf{F} = -[\mathbf{j}_d \times \mathbf{B}]c^{-1}$$

WICE instability

- Background ions can resonate with the growing waves: $k_{WICE}^{-1} \sim r_g$

- Competition between drift and ion terms

- WICE is excited when: $\left(\dfrac{n_{CR}}{n_g}\right)\left(\dfrac{v_{CR}}{v_A}\right) > \left(\dfrac{v_A}{v_{\text{th}}}\right)^2$

- Fastest growing mode: $k_{WICE} \sim \left(\dfrac{n_{CR}}{n_g}\dfrac{v_{CR}}{v_A}\dfrac{v_A^4}{v_{\text{th}}^4}\right)^{1/3} d_i^{-1} < k_{Bell}$

- Growth rate: $\gamma_{WICE} \sim \left(\dfrac{n_{CR}}{n_g}\dfrac{v_{CR}}{v_A}\dfrac{v_A}{v_{\text{th}}}\right)^{2/3} \Omega_{ci} < \gamma_{Bell}$

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	B	CR	Gas
I	Exponential growth	Unperturbed	Heating

I
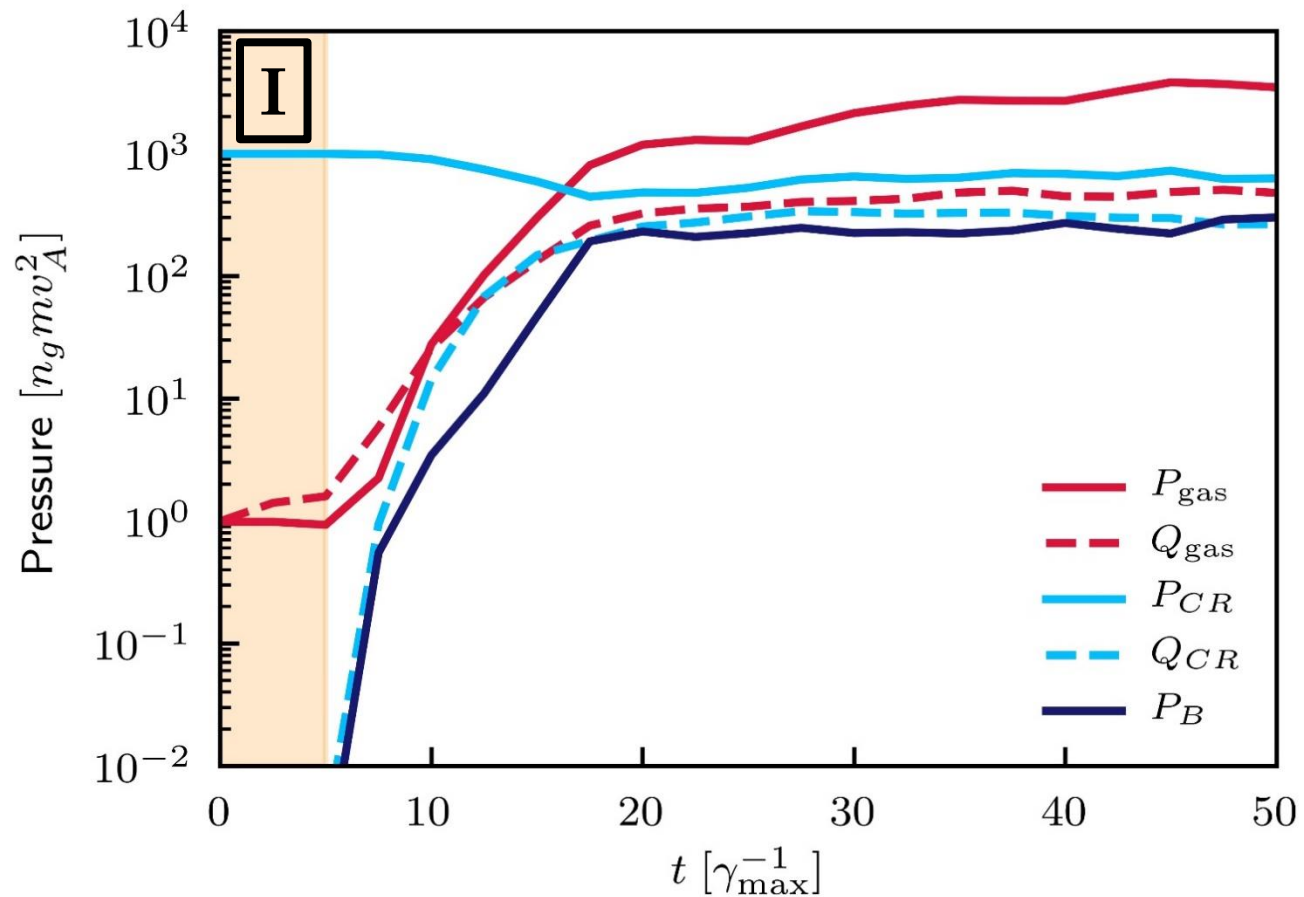
$n_{CR}/n_g = 10^{-2}$
$p_{CR}/mv_A = 10^3$
$c/v_A = 100$

Evolution of the instability (Part I)

	B	CR	Gas
I	Exponential growth	Unperturbed	Heating
II	Exponential growth	Weak scattering	Heating and acceleration
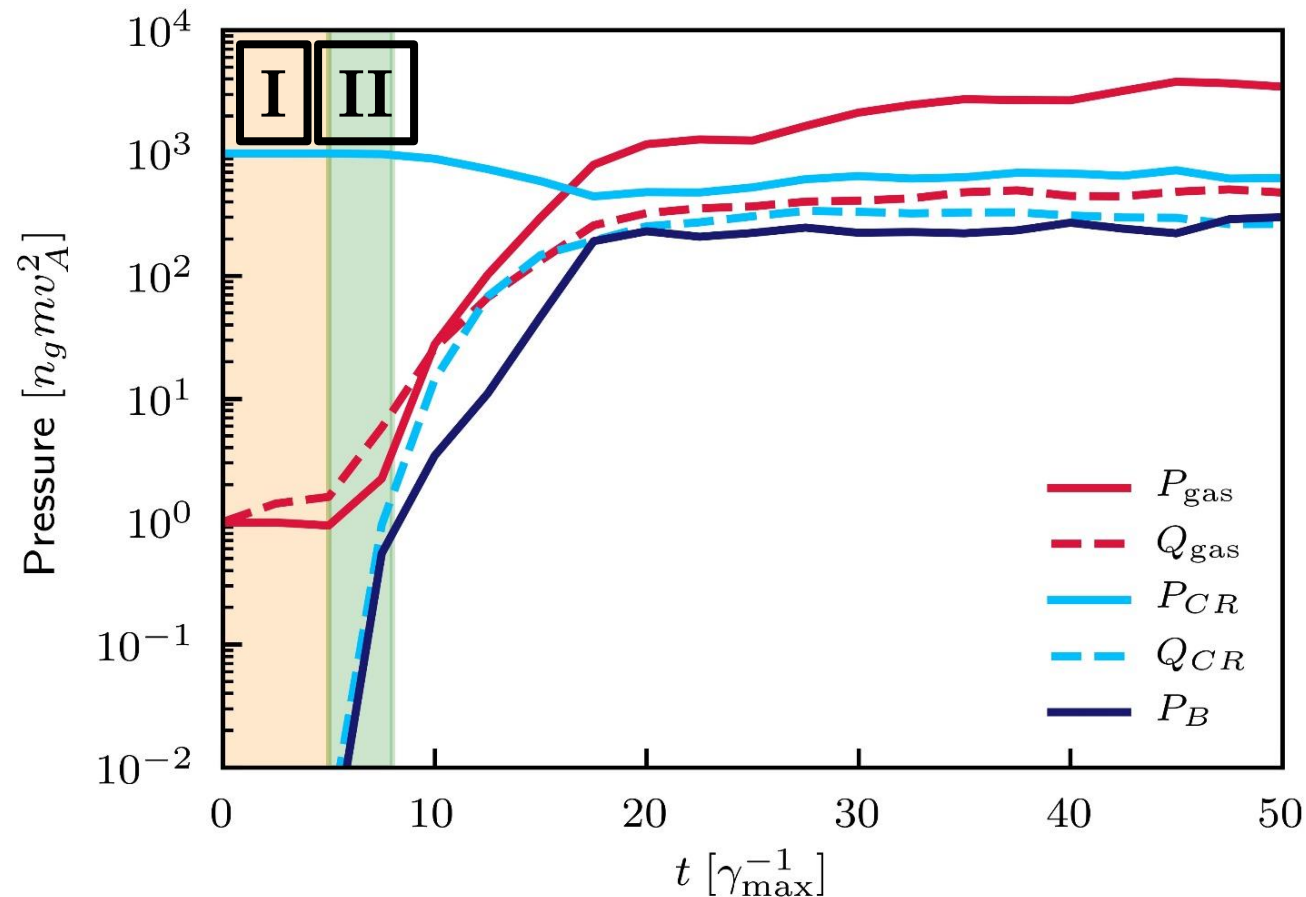
$$n_{CR}/n_g = 10^{-2}$$
$$p_{CR}/mv_A = 10^3$$
$$c/v_A = 100$$

Evolution of the instability (Part II)

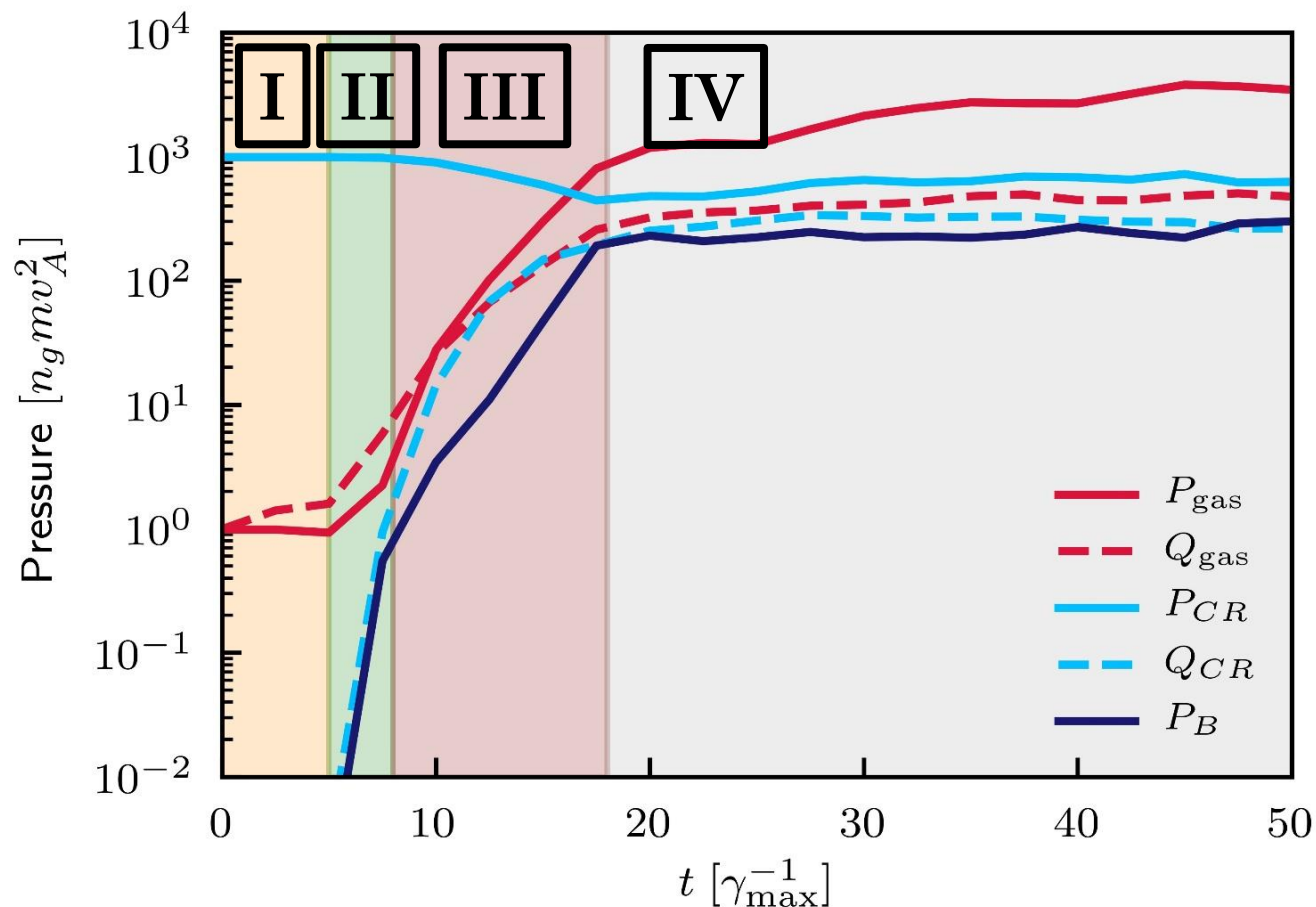|	B	CR	Gas
I	Exponential growth	Unperturbed	Heating
II	Exponential growth	Weak scattering	Heating and acceleration
III	Secular growth	Strong scattering	Weak heating and acceleration
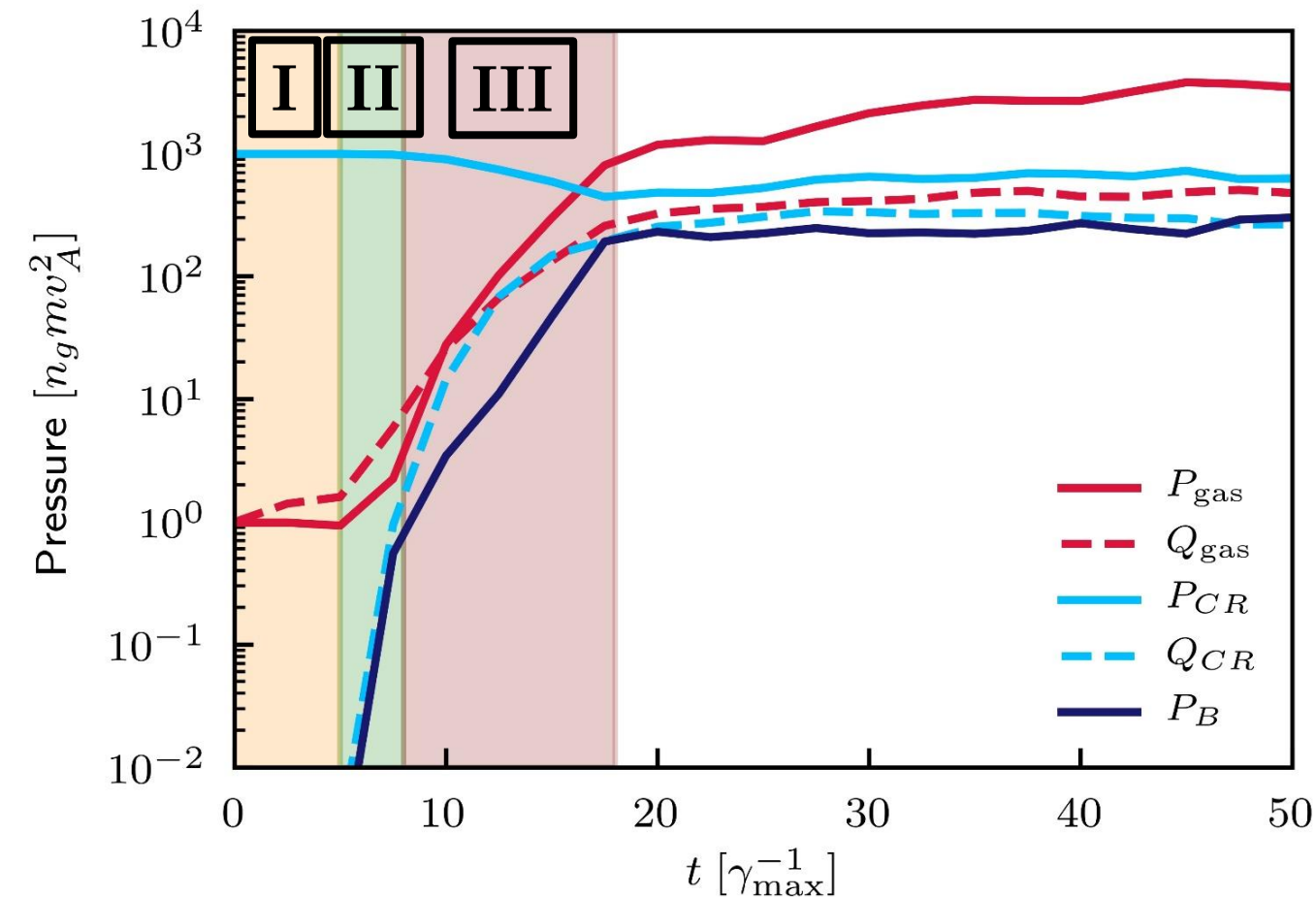
$n_{CR}/n_g = 10^{-2}$

$p_{CR}/mv_A = 10^3$

$c/v_A = 100$

Evolution of the instability (Part III)

	B	CR	Gas
I	Exponential growth	Unperturbed	Heating
II	Exponential growth	Weak scattering	Heating and acceleration
III	Secular growth	Strong scattering	Weak heating and acceleration
IV	Saturation	Isotropization	Acceleration

$n_{CR}/n_g = 10^{-2}$
$p_{CR}/mv_A = 10^3$
$c/v_A = 100$

Legend:
- P_{gas}
- Q_{gas}
- P_{CR}
- Q_{CR}
- P_B