Observation of electron rings with imaging air Cherenkov telescopes

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Motivation

- Spectroscopy of air shower electrons
  - Cherenkov light of electrons close to a telescope are imaged as rings.
  - Rings appears in CORSIKA simulation:

![Figure 1: $uv$ image of electron rings from a $\gamma$-ray shower of 10 TeV with $ELCUT(3) = 20$ MeV([5]).](image)

- $u = \sin \theta \cos \phi$
- $v = \sin \theta \sin \phi$
Cherenkov light and energy losses

Figure 2: Cherenkov emission from a muon and an electron along with the image that will be formed on the detector.

- Cherenkov emission: \( \cos \theta_c = \frac{1}{\beta n} \)
- Relevant energy loss mechanisms of electrons: Ionization and Bremsstrahlung
Figure 3: Energy loss of electrons

- For $E = E_c$ (in air, $E_c=80$ MeV; $\beta \gamma \approx 160$), $(\frac{dE}{dx})_{rad} = (\frac{dE}{dx})_{collision}$
Lorentz factor of electrons:

Analytical solution:

Figure 4: Lorentz factor of electrons as a function of emission height.
Figure 5: Emission angle of electrons with three different starting energies for two different wavelengths as a function of emission height.
Radius of electron rings ($r$)

**Figure 6:** Emission angle of electrons with three different starting energies for two different wavelengths as a function of emission height.
Number of photons per path length ($\frac{dN}{dx}$):

$$\frac{d^2N}{dx d\lambda} = \frac{2\pi \alpha z^2}{\lambda^2} \left( 1 - \frac{1}{\beta^2(x) n^2(x, \lambda)} \right) = \frac{2\pi \alpha z^2}{\lambda^2} \sin^2 \theta_c \quad (1)$$

**Figure 7:** Number of photons emitted per path length by electrons of three different starting energies.
- Photons are emitted in a cone: solid angle $\Omega$ dependence

\[
\int \frac{dN}{d\Omega dx d\lambda} d\Omega = \frac{dN}{dx d\lambda}
\]

(2)

where $d\Omega = d\phi \sin \theta d\theta$.

- Solving eqn:2 using the properties of delta function:

\[
\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\chi^2} \frac{\sin \theta(x_0)}{-d\theta(x_0) \left| \frac{dx}{dx} \right|_{x=x_0}}
\]

(3)

- Using the number of photons per solid angle ($\frac{dN}{d\Omega}$), we calculate the number of photons per typical pixel size of 0.07 degrees.
Electron rings from analytical calculation:

**Figure 8:** $\gamma_0 = 10^2$

**Figure 9:** $\gamma_0 = 1500$
Figure 10: $\gamma_0 = 10^4$

- For further details, see PoS(ICRC2019)402
Summary

- Analytical treatment of electrons; Cherenkov light emission (Ionization and Bremsstrahlung included, multiple scattering and light absorption neglected): prediction of electron ring images!
- CORSIKA simulation confirms the calculations: multiple scattering not realistically treated!(see Fig. 11 and 12)

Figure 11: Multiple scattering treated in CORSIKA vs Nature
Figure 12: $uv$ image of electron rings from a $\gamma$-ray shower of 10 TeV with $ELCUT(3) = 20$ MeV([5]).

**Future plans**

- Include multiple scattering in analytical calculation and check CORSIKA’s treatment of multiple scattering.
- Impact of local electrons on imaging techniques.
References


Energy loss mechanisms:

- Ionization energy loss: Bethe-Bloch equation
  \[ - \frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{\rho}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 \right] \] (4)

- Radiative energy loss: Bremsstrahlung
  \[ - \frac{dE}{dx} = \frac{E}{X_0} \] (5)

Constants are taken from Particle Data Group [3]

- Momentum is calculated by substituting \( E = \gamma m_e c^2 \) in eqns: 4, 5 and solving the differential equation in terms of \( \gamma \) for initial conditions \( \gamma_0 \) within a height of 500m.
Approximations for total number of particles\cite{1}:

\[ N(t) \sim \frac{0.31}{(\beta_0)^{1/2}} \exp \left[ t \left( 1 - \frac{3}{2} \ln s \right) \right] \]  \hspace{1cm} (6)

where \( t \) is the atmospheric depth in terms of radiation length \( X_0 \) \((t = X/X_0 = 21.62)\), \( X = 800\text{g/cm}^2 \) and \( s \) is the age parameter(see book\cite{1}).

**Figure 13**: Number of electrons obtained from 1TeV and 10TeV photon induced shower
Number of photons:

- Photons are emitted in a cone. Hence we need to consider the dependence of solid angle $\Omega$ while calculating the number of photons such that:

\[
\int \frac{dN}{d\Omega dx d\lambda} d\Omega = \frac{dN}{dx d\lambda}
\] (7)

where $d\Omega = d\phi \sin \theta d\theta$.

- Only way to do this is by defining a function;

\[
\frac{dN}{d\Omega dx d\lambda} = \frac{2\pi \alpha z^2}{\lambda^2 2\pi \sin \theta} \sin^2 \theta_c \delta(\theta - \theta_c(x))
\] (8)

For our convenience in calculation, lets fix wavelength $\lambda = 500$nm. Then eqn: 7 can be written as:

\[
\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\lambda^2} \int dx \sin \theta ch \delta(\theta - \theta_c(x))
\] (9)
Eqn: 9 can be solved using the derivative and integral properties of delta function which are given below:

\[
\delta(g(x)) = \sum_{i=1}^{N} \frac{1}{|g'(x_i)|} \delta(x - x_i) \tag{10}
\]

\[
\int f(x) \delta(g(x)) dx = \sum_{i=1}^{N} \frac{f(x_i)}{|g'(x_i)|} \tag{11}
\]

where \( f(x) = \sin \theta_c, \ g(x) = (\theta - \theta_c(x)) \) and \( N \) is the total number of roots by which \( dN/d\Omega \) can be solved as:

\[
\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\lambda^2} \left| \frac{-d\theta(x_0)}{dx} \right| \bigg|_{x=x_0} \sin \theta(x_0) \tag{12}
\]
At the end of the calculation to make $\frac{dN}{d\Omega}$ dimensionally correct, we have multiplied the number with $\lambda = 100\,\text{nm}$ (mathematically there is $\lambda$ dependence in the denominator of LHS in eqn: 7). Using the number of photons per solid angle, we calculate the number of photons per typical pixel size of $1.33\,\text{mrad}$ (for HESS Phase II) and used it to reproduce the rings.

**Multiple scattering**

Average angle of deflection by multiple scattering:

$$\sqrt{\langle \theta^2 \rangle} = \frac{13.6\,\text{MeV}}{\beta c p} \sqrt{z \frac{X}{X_0}} \left( 1 + 0.038 \ln \frac{X}{X_0} \right) \quad (13)$$

where $p$, $c$ and $z$ are the momentum, velocity, and charge number of the incident particle, and $X/X_0$ is the thickness of the scattering medium in radiation lengths. $\theta$ was coming to be very big (around $15^0$) which was not appearing in the $uv$ image from CORSIKA.