Observation of electron rings with imaging air Cherenkov telescopes

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Motivation

- Spectroscopy of air shower electrons
  - Cherenkov light of electrons close to a telescope are imaged as rings.
  - Rings appears in CORSIKA simulation:

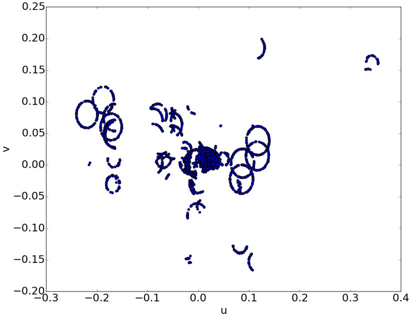

Figure 1: uv image of electron rings from a γ-ray shower of 10 TeV with $ELCUT(3) = 20$ MeV([5]).

- $u = \sin\theta\cos\phi$
- $v = \sin\theta\sin\phi$

Cherenkov light and energy losses

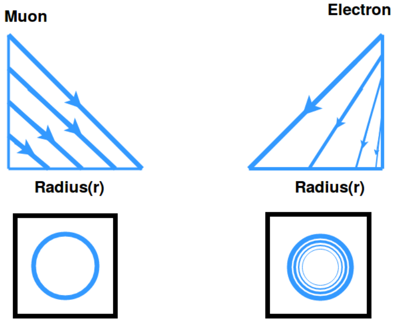

Figure 2: Cherenkov emission from a muon and an electron along with the image that will be formed on the detector.

- Cherenkov emission: $\cos\theta_c = \frac{1}{\beta n}$
- Relevant energy loss mechanisms of electrons: Ionization and Bremsstrahlung

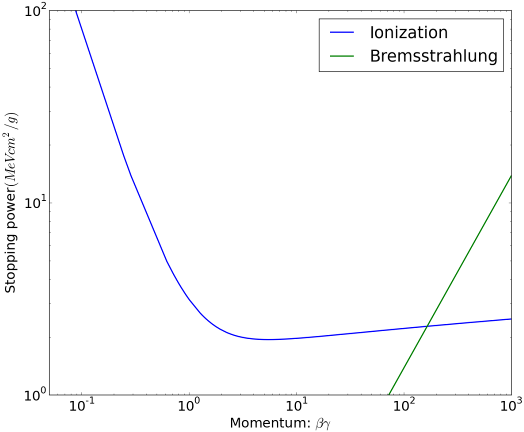

Figure 3: Energy loss of electrons

- For $E = E_c$(in air, E_c=80MeV; $\beta\gamma \approx 160$), $\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{collision}$

Lorentz factor of electrons:

Analytical solution:

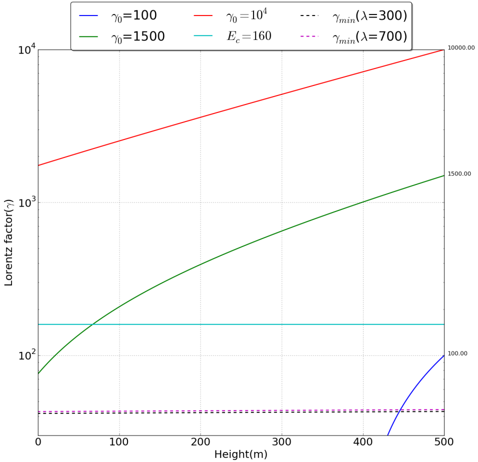

Figure 4: Lorentz factor of electrons as a function of emission height.

Angle of emission(θ_c):

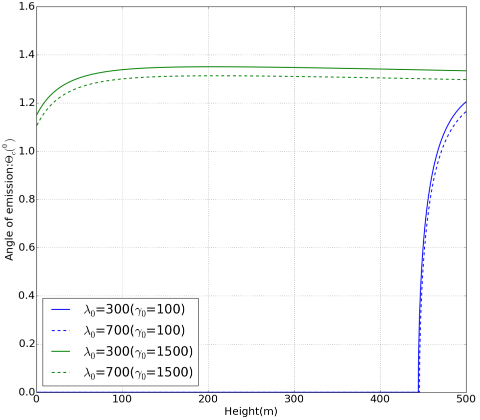

Figure 5: Emission angle of electrons with three different starting energies for two different wavelengths as a function of emission height.

Radius of electron rings(r)

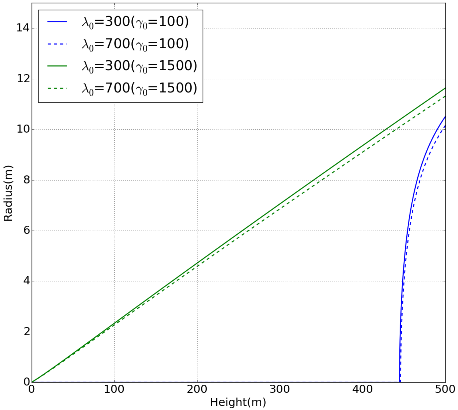

Figure 6: Emission angle of electrons with three different starting energies for two different wavelengths as a function of emission height.

Number of photons per path length($\frac{dN}{dx}$):

$$\frac{d^2N}{dxd\lambda} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2(x)n^2(x,\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_c \qquad (1)$$

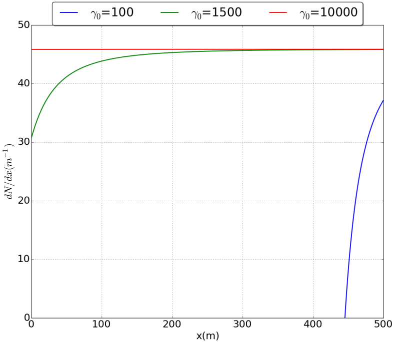

Figure 7: Number of photons emitted per path length by electrons of three different starting energies.

- Photons are emitted in a cone: solid angle Ω dependence

$$\int \frac{dN}{d\Omega dx d\lambda} d\Omega = \frac{dN}{dx d\lambda} \tag{2}$$

where $d\Omega = d\phi \sin\theta d\theta$.

- Solving eqn:2 using the properties of delta function:

$$\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\lambda^2} \frac{\sin\theta(x_0)}{\left|\frac{-d\theta(x_0)}{dx}\right|_{x=x_0}} \tag{3}$$

- Using the number of photons per solid angle $(\frac{dN}{d\Omega})$, we calculate the number of photons per typical pixel size of 0.07 degrees.

Electron rings from analytical calculation:

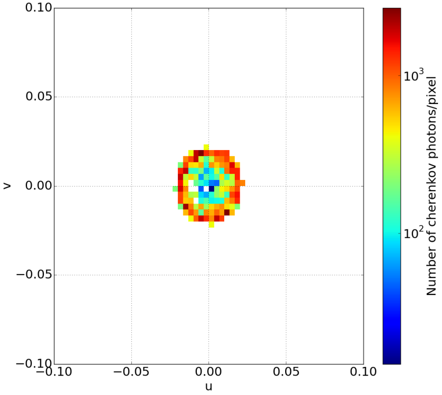

Figure 8: $\gamma_0 = 10^2$

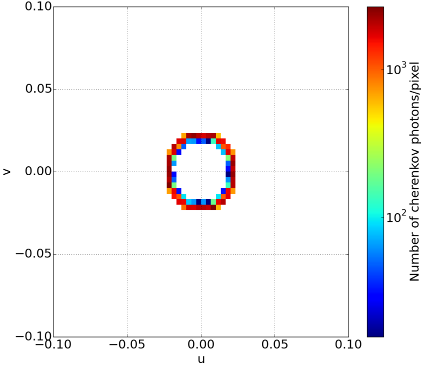

Figure 9: $\gamma_0 = 1500$

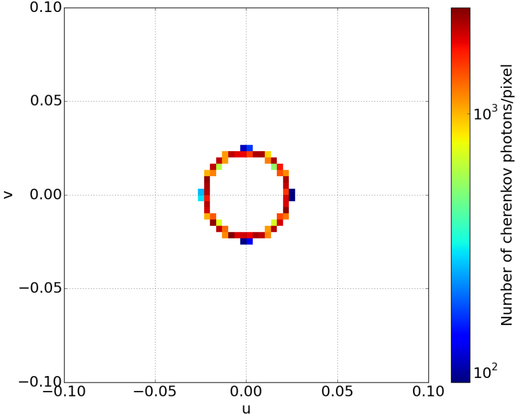

Figure 10: $\gamma_0 = 10^4$

- For further details, see PoS(ICRC2019)402

Summary

- Analytical treatment of electrons; Cherenkov light emission (Ionization and Bremsstrahlung included, multiple scattering and light absorption neglected): prediction of electron ring images!
- CORSIKA simulation confirms the calculations: multiple scattering not realistically treated!(see Fig. 11 and 12)

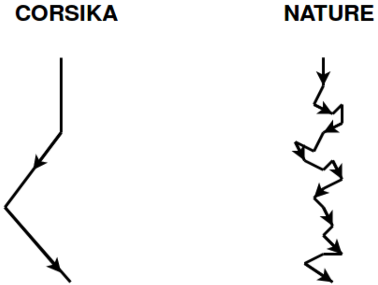

Figure 11: Multiple scattering treated in CORSIKA vs Nature

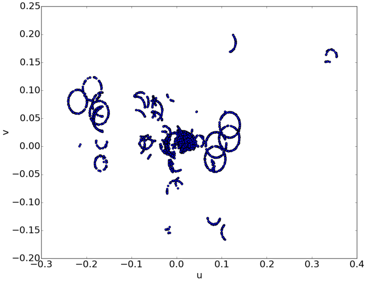

Figure 12: uv image of electron rings from a γ-ray shower of 10 TeV with $ELCUT(3) = 20$ MeV([5]).

Future plans

- Include multiple scattering in analytical calculation and check CORSIKA's treatment of multiple scattering.
- Impact of local electrons on imaging techniques.

References

[1] Gaisser, Thomas K. and Engel, Ralph and Resconi, Elisa, *Cosmic Rays and Particle Physics*, Cambridge University Press, Cambridge 2016

[2] Greisen, K., *Prog. Cosmic Ray Physics*, **Volume3** (1956) 1

[3] Patrignani, C. and others, *Review of Particle Physics*, *Particle Data Group*, Chin. Phys., **Volume C40** 2016

[4] Philip E. Ciddor, *Refractive index of air: new equations for the visible and near infrared*, Appl. Opt. 35, **1566-1573** (1996)

[5] D.Heck et al., Report **FZKA 6019** (1998), Forschungszentrum Karlsruhe; http://www-ik.fzk.de/corsika/physics-description/corsika-phys.html

Energy loss mechanisms:

- Ionization energy loss: Bethe-Bloch equation

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{\rho}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{max}}{I^2} - \beta^2\right] \tag{4}$$

- Radiative energy loss: Bremsstrahlung

$$-\frac{dE}{dx} = \frac{E}{X_0} \tag{5}$$

  constants are taken from Particle Data Group [3]

- Momentum is calculated by substituting $E = \gamma m_e c^2$ in eqns: 4, 5 and solving the differential equation in terms of γ for initial conditions γ_0 within a height of 500m.

- Approximations for total number of particles[1]:

$$N(t) \sim \frac{0.31}{(\beta_0)^{1/2}} \exp\left[t\left(1 - \frac{3}{2}\ln s\right)\right] \tag{6}$$

where t is the atmospheric depth in terms of radiation length X_0 ($t = X/X_0 = 21.62$), $X = 800\text{g/cm}^2$ and s is the age parameter(see book [1]).

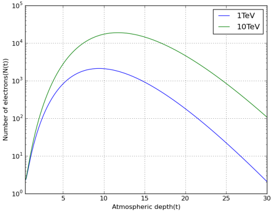

Figure 13: Number of electrons obtained from 1TeV and 10TeV photon induced shower

Number of photons:

- Photons are emitted in a cone. Hence we need to consider the dependence of solid angle Ω while calculating the number of photons such that:

$$\int \frac{dN}{d\Omega dx d\lambda} d\Omega = \frac{dN}{dx d\lambda} \tag{7}$$

where $d\Omega = d\phi \sin\theta d\theta$.

- Only way to do this is by defining a function;

$$\frac{dN}{d\Omega dx d\lambda} = \frac{2\pi\alpha z^2}{\lambda^2 2\pi \sin\theta} \sin^2\theta_c \delta(\theta - \theta_c(x)) \tag{8}$$

For our convenience in calculation, lets fix wavelength $\lambda = 500$nm. Then eqn: 7 can be written as:

$$\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\lambda^2} \int dx \sin\theta_{ch} \delta(\theta - \theta_c(x)) \tag{9}$$

Eqn: 9 can be solved using the derivative and integral properties of delta function which are given below:

$$\delta(g(x)) = \sum_{i=1}^{N} \frac{1}{|g'(x_i)|}\delta(x - x_i) \tag{10}$$

$$\int f(x)\delta(g(x))dx = \sum_{i=1}^{N} \frac{f(x_i)}{|g'(x_i)|} \tag{11}$$

where $f(x) = \sin\theta_c$, $g(x) = (\theta - \theta_c(x))$ and N is the total number of roots by which $dN/d\Omega$ can be solved as:

$$\frac{dN}{d\lambda d\Omega} = \frac{\alpha z^2}{\lambda^2} \frac{\sin\theta(x_0)}{\left|\frac{-d\theta(x_0)}{dx}\right|_{x=x_0}} \tag{12}$$

- At the end of the calculation to make $\frac{dN}{d\Omega}$ dimensionally correct, we have multiplied the number with $\lambda = 100$nm (mathematically there is λ dependence in the denominator of LHS in eqn: 7). Using the number of photons per solid angle, we calculate the number of photons per typical pixel size of 1.33mrad(for HESS Phase II) and used it to reproduce the rings.

Multiple scattering

Average angle of deflection by multiple scattering:

$$\sqrt{<\theta^2>} = \frac{13.6MeV}{\beta cp} z\sqrt{X/X_0}(1 + 0.038\ln X/X_0) \quad (13)$$

where p, c and z are the momentum, velocity, and charge number of the incident particle, and X/X_0 is the thickness of the scattering medium in radiation lengths. θ was coming to be very big (around 15^0) which was not appearing in the uv image from CORSIKA.