Solar Modulation of Cosmic Rays in a Semi-Analytical Framework

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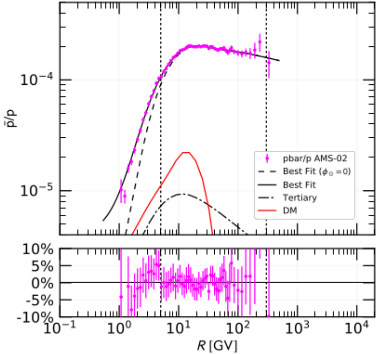
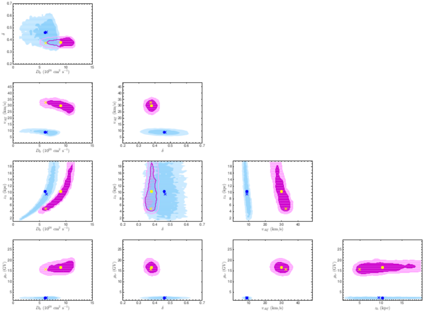
Importance of Solar Modulation



Understand cosmic ray fluxes at low energies to interpret potential dark matter signals.

Cuoco et al. arXiv:1903.01472



Disentangle modulation of galactic cosmic rays from processes in the heliosphere.

Johannesson et al. arXiv:1602.02243

Solving the Transport Equation

computational expense →

Force-field

- ✓ fast
- ✗ inaccurate
- ✗ local

Gleeson & Axford 1968

Numerical codes

- ✗ slow
- ✓ accurate
- ✓ global

Aslam et al., arXiv:1811.10710, Boschini et al., arXiv:1704.03733, Vittino et al., arXiv:1707.09003,

Kappl, arXiv:1601.02832

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Force Field Approximation

Gleeson & Axford, Caballero-Lopez & Moraal

Rewrite the transport equation as

$$\frac{\partial f}{\partial t} + \nabla \cdot (C\mathbf{V}f - \mathbf{K} \cdot \nabla f) + \frac{1}{3p^2}\frac{\partial}{\partial p}(p^3\mathbf{V} \cdot \nabla f) = Q\,,$$

with Compton Getting factor $C \equiv -\frac{p}{3}\frac{1}{f}\frac{\partial f}{\partial p}$.

Assumptions:

- Steady state, $\partial f/\partial t = 0$
- No sources, $Q = 0$
- No average momentum loss in lab frame, $\langle \dot{p} \rangle = \frac{1}{3}\mathbf{V} \cdot \nabla f/f = 0$

Zero streaming condition:

$$C\mathbf{V}f - \mathbf{K} \cdot \nabla f = 0.$$

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Force Field Solution

Gleeson & Axford, Caballero-Lopez & Moraal

Assuming spherical symmetry:

$$\frac{\partial f}{\partial r} + \frac{Vp}{3\kappa}\frac{\partial f}{\partial p} = 0,$$

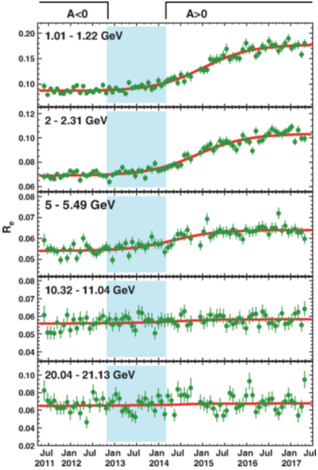
Method of characteristics:

$$\int_{p_{TOA}}^{p_{LIS}} \frac{\beta \kappa_{p'}}{p'} dp' = \int_{r_{TOA}}^{r_{LIS}} \frac{V}{3\kappa_{r'}} dr' \equiv \phi(r),$$

For $\kappa_p \propto p$ and $\beta \approx 1 \longrightarrow \phi = p_{LIS} - p_{TOA}$.
Conservation of the phase-space density f leads to:

$$\frac{J_{TOA}}{p_{TOA}^2} = \frac{J_{LIS}}{p_{LIS}^2}.$$

Time-Dependent Experimental Data



AMS Collaboration

Phys. Rev. Lett. 121, 051102

$$R_e = \frac{\Phi_{e^+}}{\Phi_{e^-}}$$

Explanation of current data requires charge sign dependent effects.
$\Longrightarrow$ Importance of drifts.

To explain the AMS-02 data we make modifications to the force field model.

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Changes to Force Field Model

Starting again from divergence free streaming

$$\int_S (C\mathbf{V}f - \mathbf{K}\cdot\nabla f)\cdot d\mathbf{S} = 0$$

Solve the transport equation in 2D including gradient curvature drifts.

Where we introduce angular averages

$$\tilde{f} = \int_0^{\pi/2} \mathrm{d}\theta \sin\theta f$$

$$\tilde{V} = \left(\frac{\partial\tilde{f}}{\partial p}\right)^{-1} \int_0^{\pi/2} \mathrm{d}\theta \sin\theta\, V\,\partial_p f$$

$$\tilde{K}_{rr} = \left(\frac{\partial\tilde{f}}{\partial r}\right)^{-1} \int_0^{\pi/2} \mathrm{d}\theta \sin\theta\, K_{rr}\partial_r f$$

$$\tilde{v}_{gc,r} = \left(\tilde{f}\right)^{-1} \int_0^{\pi/2} \mathrm{d}\theta \sin\theta\, v_{gc,r} f$$

Changes to Force Field Model

After angular averages this reduces to

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p\,\tilde{V}}{3\tilde{K}_{rr}}\frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}}\tilde{f}$$

Can be solved using the method of characteristics

$$\tilde{f}(r,p) = f_{\mathrm{LIS}}(p_{\mathrm{LIS}})\, \mathrm{e}^{-\int_0^r \mathrm{d}r' \frac{\tilde{v}_{gc,r}(r',p'_{\mathrm{LIS}})}{K_{rr}(r',p'_{\mathrm{LIS}})}}$$

With $p_{\mathrm{LIS}}(r,p)$ the solution to the initial value problem

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{p\,\tilde{V}}{3\tilde{K}_{rr}}\,,$$

with $p_{\mathrm{LIS}}(R,p) = p$.

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$$\tilde{f}(r,p) = f_{\mathrm{LIS}}(p_{\mathrm{LIS}})\,\mathrm{e}^{-\int_0^r \mathrm{d}r' \frac{\tilde{v}_{gc,r}(r',p'_{\mathrm{LIS}})}{K_{rr}(r',p'_{\mathrm{LIS}})}}$$

We parametrize them as

$$\tilde{V} = V_0(1 + \Delta V\,\theta(p - p_b))$$

$$\tilde{K}_{rr} = K_0 R^a \left(\frac{R^c + R_k^c}{1 + R_k^c}\right)^{(b-a)/c}$$

$$\tilde{v}_{gc,r} = \kappa_0 \frac{\beta p}{3B_0}\frac{10\,p^2}{1 + 10\,p^2}$$

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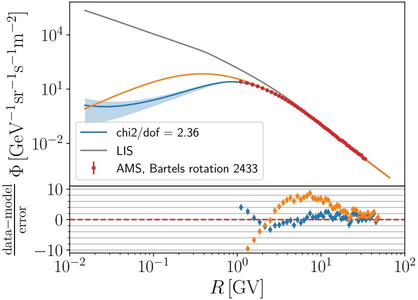
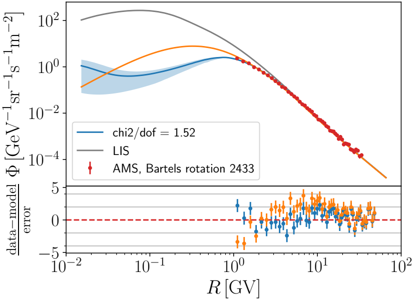
Example: Fit to AMS-02 Data

Bartels Rotation 2433

Electrons



Positrons

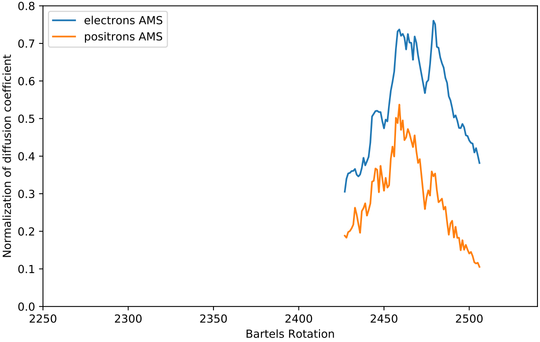


Can explain data accurately while the conventional force field model fails.

LIS from Vittino et al. arXiv:1904.05899

Correlation with Solar Wind Parameters

We find a correlation between the tilt angle and the diffusion coefficient.



$$\tilde{K}_{rr}^{-} = K_0^{-} R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c}$$

$$\tilde{K}_{rr}^{+} = K_0^{+} R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c}$$

Tilt angle from http://wso.stanford.edu/

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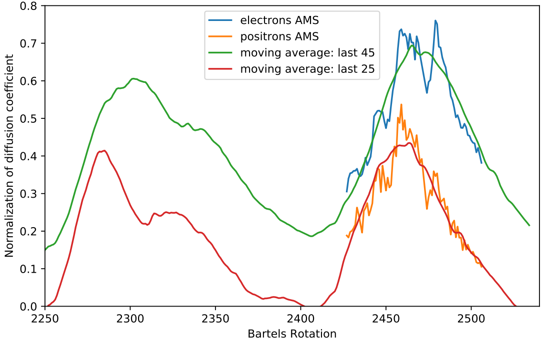

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$$K_0^{-} = a^{-} \langle \alpha \rangle_{\Delta t} + b^{-}$$

$$\tilde{K}_{rr}^{+} = K_0^{+} R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c}$$

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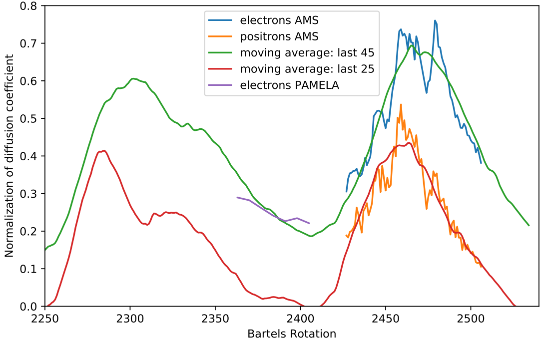

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$$K_0^{+} = a^{+} \langle \alpha \rangle_{\Delta t} + b^{+}$$

$$\tilde{K}_{rr}^{-} = K_0^{-} R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c}$$

Tilt angle from http://wso.stanford.edu/

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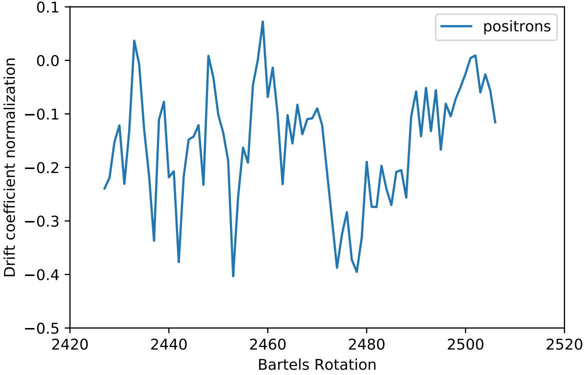
Correlation with Solar Wind Parameters

We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.



$$\tilde{v}^{+}_{gc,r} = \kappa^{+}_{0} \frac{\beta p}{3 B_0} \frac{10\, p^2}{1 + 10\, p^2}$$

Magnetic field strength from http://www.srl.caltech.edu/ACE/

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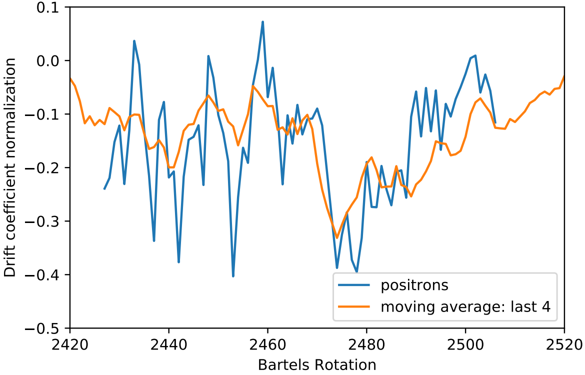

$$\tilde{v}^{+}_{gc,r} = \kappa^{+}_{0} \frac{\beta p}{3 B_0} \frac{10\, p^2}{1 + 10\, p^2}$$

$$\kappa^{+}_{0} = c^{+}_{0} \langle B_0 \rangle_{\Delta t} + d^{+}_{0}$$

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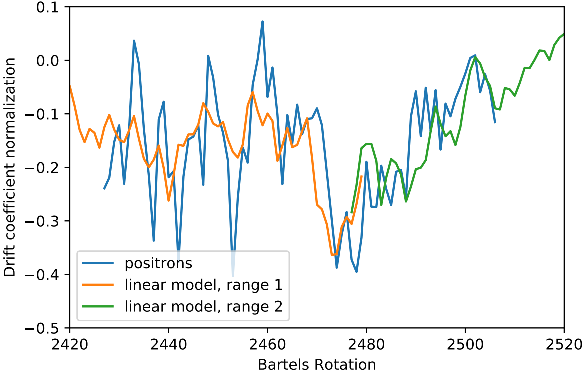

$$\tilde{v}^{+}_{gc,r} = \kappa^{+}_{0} \frac{\beta p}{3 B_0} \frac{10\, p^2}{1 + 10\, p^2}$$

$$\kappa^{+}_{0} = c^{+}_{0} \langle B_0 \rangle_{\Delta t} + d^{+}_{0}$$

$$\kappa^{+}_{0} = c^{+}_{1} \langle B_0 \rangle_{\Delta t} + d^{+}_{1}$$

Magnetic field strength from http://www.srl.caltech.edu/ACE/

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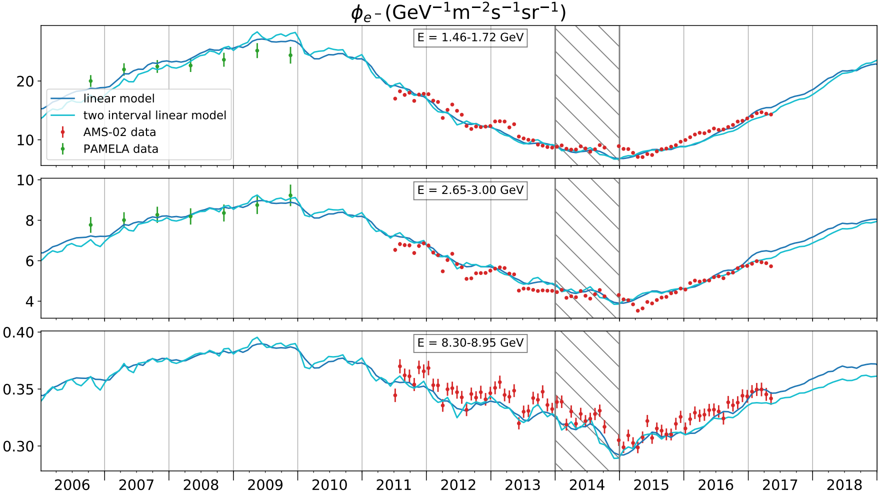
Prediction of Electron Flux



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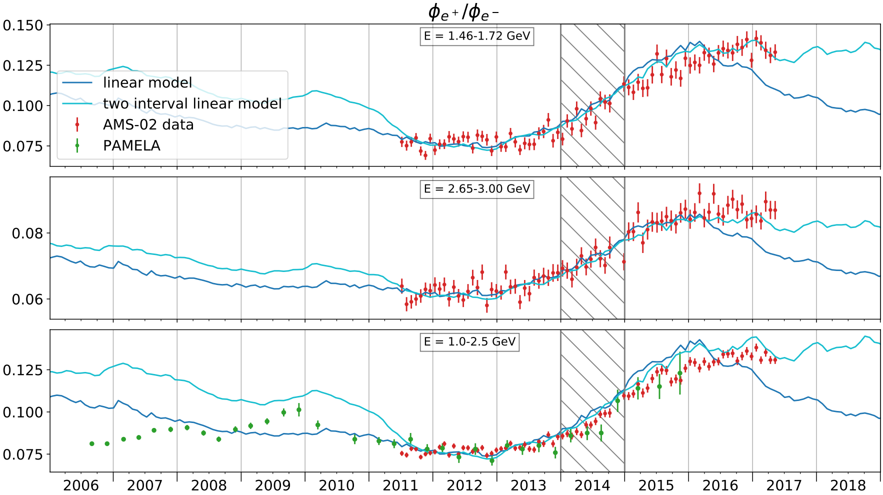
Prediction of Positron Ratio



Conclusion

- We have developed a semi analytical method to solve the 2D transport equation.
- Our method runs significantly faster than fully numerical model (~ 20 ms).
- We are able to reproduce AMS-02 electron and positron fluxes.

Thank you for your attention!

Download our code at
https://git.rwth-aachen.de/kuhlenmarco/effmod-code