Solar Modulation of Cosmic Rays in a Semi-Analytical Framework

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Understand cosmic ray fluxes at low energies to interpret potential dark matter signals.

Cuoco et al. arXiv:1903.01472

Disentangle modulation of galactic cosmic rays from processes in the heliosphere.

Johannesson et al. arXiv:1602.02243
Solving the Transport Equation

computational expense

Force-field
✓ fast
✗ inaccurate
✗ local

Gleeson & Axford 1968

Numerical codes
✗ slow
✓ accurate
✓ global

Aslam et al., arXiv:1811.10710,
Boschini et al., arXiv:1704.03733,
Vittino et al., arXiv:1707.09003,
Kappl, arXiv:1601.02832
### Computational Expense

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<th>Method</th>
<th>Fast</th>
<th>Accurate</th>
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<tr>
<td>Force-field</td>
<td>✓</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Semi-analytical</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>Numerical codes</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
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</table>

- Force-field: **✓** fast, **X** inaccurate, **X** local
- Semi-analytical: **✓** fast, **✓** accurate, **X** local
- Numerical codes: **X** slow, **✓** accurate, **✓** global

Gleeson & Axford 1968

Rewrite the transport equation as
\[ \frac{\partial f}{\partial t} + \nabla \cdot \left( CVf - K \cdot \nabla f \right) + \frac{1}{3p^2} \frac{\partial}{\partial p} \left( p^3 V \cdot \nabla f \right) = Q, \]
with Compton Getting factor
\[ C \equiv -\frac{p}{3f} \frac{\partial f}{\partial p}. \]

Assumptions:
- Steady state, \( \frac{\partial f}{\partial t} = 0 \)
- No sources, \( Q = 0 \)
- No average momentum loss in lab frame, \( \langle \dot{p} \rangle = \frac{1}{3} V \cdot \nabla f / f = 0 \)

Zero streaming condition:
\[ CVf - K \cdot \nabla f = 0. \]
Assuming spherical symmetry:

\[
\frac{\partial f}{\partial r} + \frac{Vp}{3\kappa} \frac{\partial f}{\partial p} = 0,
\]

Method of characteristics:

\[
\int_{p_{TOA}}^{p_{LIS}} \frac{\beta \kappa_p'}{p'} dp' = \int_{r_{TOA}}^{r_{LIS}} \frac{V}{3\kappa_{r'}} dr' \equiv \phi(r),
\]

For \( \kappa_p \propto p \) and \( \beta \approx 1 \) \( \rightarrow \) \( \phi = p_{LIS} - p_{TOA} \).

Conservation of the phase-space density \( f \) leads to:

\[
\frac{J_{TOA}}{p_{TOA}^2} = \frac{J_{LIS}}{p_{LIS}^2}.
\]
Explanation of current data requires charge sign dependent effects.

$R_e = \frac{\Phi_{e^+}}{\Phi_{e^-}}$

Importance of drifts.

To explain the AMS-02 data we make modifications to the force field model.
Starting again from divergence free streaming:

\[ \int_S (C \mathbf{V} f - \mathbf{K} \cdot \nabla f) \cdot d\mathbf{S} = 0 \]

Solve the transport equation in 2D including gradient curvature drifts.

Where we introduce angular averages:

\[ \tilde{f} = \int_0^{\pi/2} d\theta \sin \theta f \]

\[ \tilde{V} = \left( \frac{\partial \tilde{f}}{\partial p} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta V \partial_p f \]

\[ \tilde{K}_{rr} = \left( \frac{\partial \tilde{f}}{\partial r} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta K_{rr} \partial_r f \]

\[ \tilde{v}_{gc,r} = \left( \tilde{f} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta v_{gc,r} f \]
After angular averages this reduces to

\[
\frac{\partial \tilde{f}}{\partial r} + \frac{p \tilde{V}}{3 \tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}
\]

Can be solved using the method of characteristics

\[
\tilde{f}(r, p) = f_{\text{LIS}}(p_{\text{LIS}})
\]

\[
e^{-\int_0^r dr' \frac{\tilde{v}_{gc,r}(r', p'_{\text{LIS}})}{\tilde{K}_{rr}(r', p'_{\text{LIS}})}}
\]

With \(p_{\text{LIS}}(r, p)\) the solution to the initial value problem

\[
\frac{dp}{dr} = \frac{p \tilde{V}}{3 \tilde{K}_{rr}},
\]

with \(p_{\text{LIS}}(R, p) = p\).
After angular averages this reduces to

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p \tilde{V}}{3 \tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}$$

Can be solved using the method of characteristics

$$\tilde{f}(r, p) = f_{\text{LIS}}(p_{\text{LIS}}) e^{-\int_0^r dr' \tilde{v}_{gc,r}(r', p'_{\text{LIS}})/\tilde{K}_{rr}(r', p'_{\text{LIS}})}$$

We parametrize them as

$$\tilde{V} = V_0 (1 + \Delta V \theta(p - p_b))$$

$$\tilde{K}_{rr} = K_0 R^a \left( \frac{R^c + R^c_k}{1 + R^c_k} \right)^{(b-a)/c}$$

$$\tilde{v}_{gc,r} = \kappa_0 \frac{\beta p}{3 B_0} \frac{10 p^2}{1 + 10 p^2}$$
Example: Fit to AMS-02 Data

Electrons

Bartels Rotation 2433

Positrons

Can explain data accurately while the conventional force field model fails.

LIS from Vittino et al. arXiv:1904.05899
We find a correlation between the tilt angle and the diffusion coefficient.

\[ \tilde{K}_{rr}^- = K_0^- R^a \left( \frac{R_c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \]

\[ \tilde{K}_{rr}^+ = K_0^+ R^a \left( \frac{R_c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \]

Tilt angle from http://wso.stanford.edu/
We find a correlation between the tilt angle and the diffusion coefficient. 

\[
\tilde{K}_{rr}^- = K_0^- R^a \left( \frac{R_c^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \\
K_0^- = a^- \langle \alpha \rangle \Delta t + b^- \\
\tilde{K}_{rr}^+ = K_0^+ R^a \left( \frac{R_c^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \\
K_0^+ = a^+ \langle \alpha \rangle \Delta t + b^+
\]

Tilt angle from http://wso.stanford.edu/
We find a correlation between the tilt angle and the diffusion coefficient.

\[ \tilde{K}_{rr}^- = K_0^- R^a \left( \frac{R_c^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \]

\[ K_0^- = a^- \langle \alpha \rangle \Delta t + b^- \]

\[ \tilde{K}_{rr}^+ = K_0^+ R^a \left( \frac{R_c^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} \]

\[ K_0^+ = a^+ \langle \alpha \rangle \Delta t + b^+ \]

Tilt angle from http://wso.stanford.edu/
We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.

\[ \tilde{v}_{gc,r} = \kappa_0^+ \frac{\beta p}{3B_0} \frac{10p^2}{1 + 10p^2} \]

Magnetic field strength from http://www.srl.caltech.edu/ACE/
We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.

\[
\hat{v}_{gc,r} = \kappa_0^+ \frac{\beta p}{3 B_0} \frac{10 p^2}{1 + 10 p^2}
\]

\[
\kappa_0^+ = c_0^+ \langle B_0 \rangle \Delta t + d_0^+
\]

Magnetic field strength from http://www.srl.caltech.edu/ACE/
We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.

\[ \tilde{v}_{gc,r}^+ = \kappa_0^+ \frac{\beta p}{3B_0} \frac{10 p^2}{1 + 10 p^2} \]

\[ \kappa_0^+ = c_0^+ \langle B_0 \rangle \Delta t + d_0^+ \]

\[ \kappa_0^+ = c_1^+ \langle B_0 \rangle \Delta t + d_1^+ \]

Magnetic field strength from http://www.srl.caltech.edu/ACE/
Comparing to Data - Prediction of Fluxes

Prediction of Electron Flux

$\phi_e^-(\text{GeV}^{-1}\text{m}^{-2}\text{s}^{-1}\text{sr}^{-1})$

- Linear model
- Two interval linear model
- AMS-02 data
- PAMELA data

E = 1.46-1.72 GeV
E = 2.65-3.00 GeV
E = 8.30-8.95 GeV

Prediction of Positron Ratio

\( \phi_{e^+}/\phi_{e^-} \)

- Linear model
- Two interval linear model
- AMS-02 data
- PAMELA

Comparing to Data - Prediction of Fluxes

\( E = 1.46-1.72 \text{ GeV} \)

\( E = 2.65-3.00 \text{ GeV} \)

\( E = 1.0-2.5 \text{ GeV} \)
We have developed a semi analytical method to solve the 2D transport equation.

Our method runs significantly faster than fully numerical model ($\sim 20$ ms).

We are able to reproduce AMS-02 electron and positron fluxes.
Thank you for your attention!

Download our code at
https://git.rwth-aachen.de/kuhlenmarco/effmod-code