

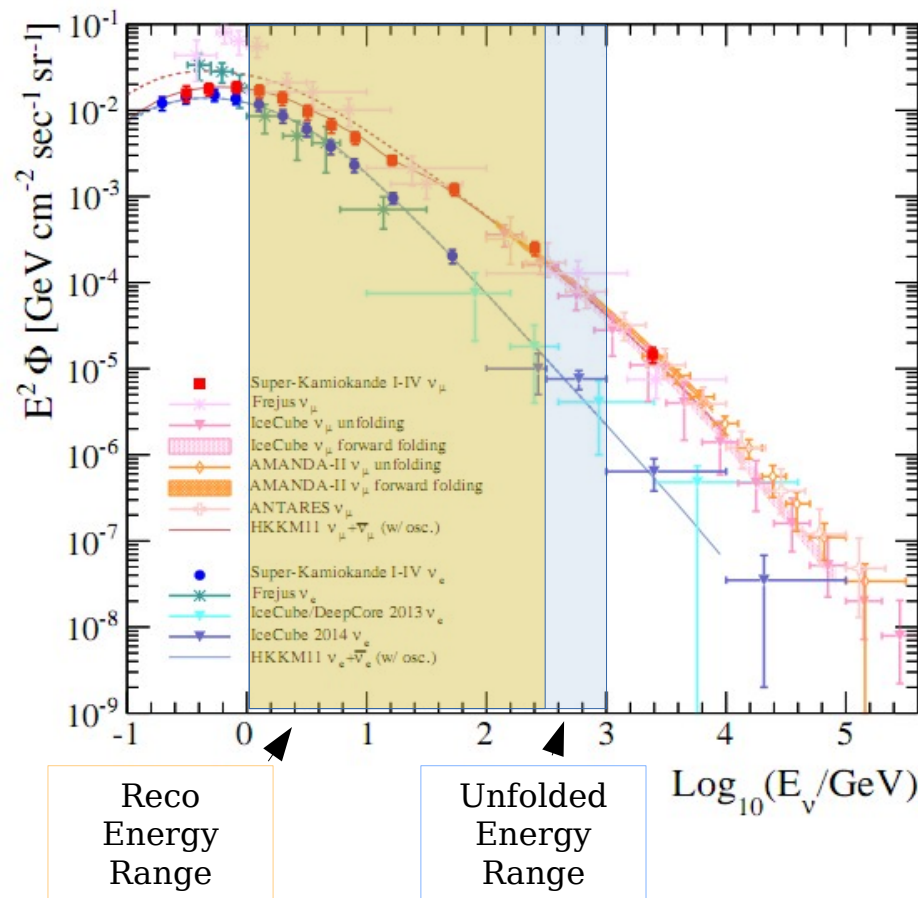
Atmospheric Neutrino Unfolding

Joakim Sandroos, J. G.U. Mainz

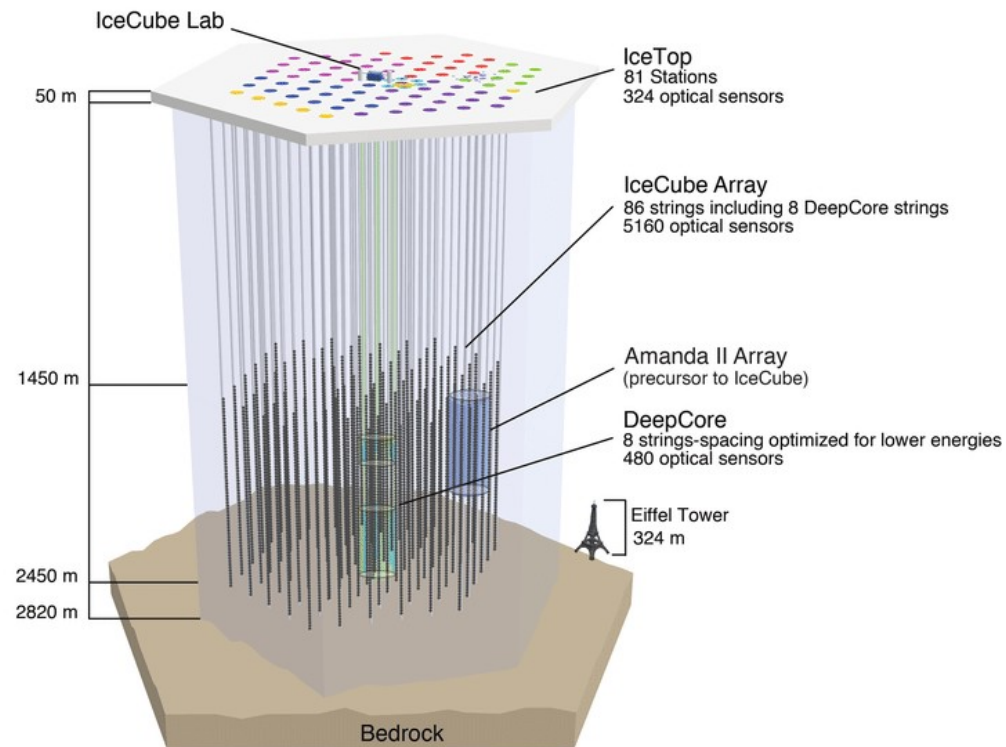
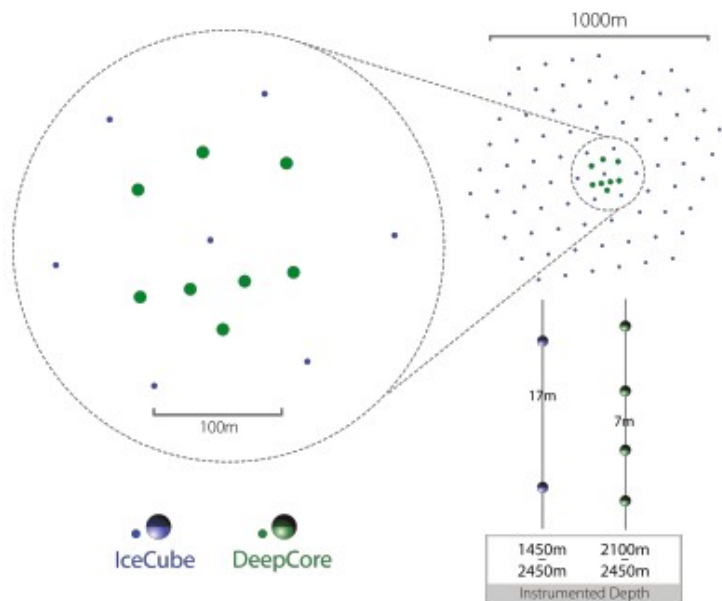
for the IceCube collaboration



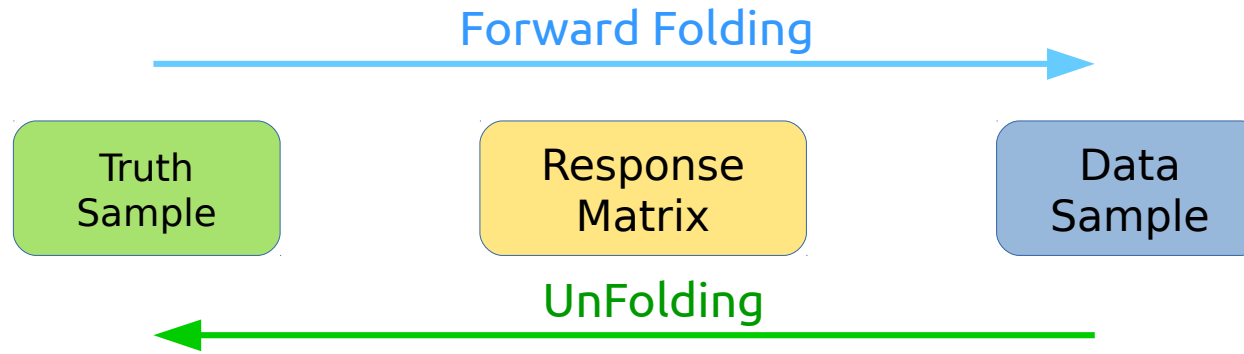
- Goal: Provide model agnostic 'true' data sample for model builders to test against
- Goal: Accurate 'beam' measurement for any experiment depending on atmospheric neutrinos
- Means: Model agnostic Unfolding
- Blind: Analysis tested on MC and 10% data sample.



- Antarctica
- 1km³ instrumented ice
- Cherenkov radiation
- 5160 optical modules
- Outer detector as muon veto



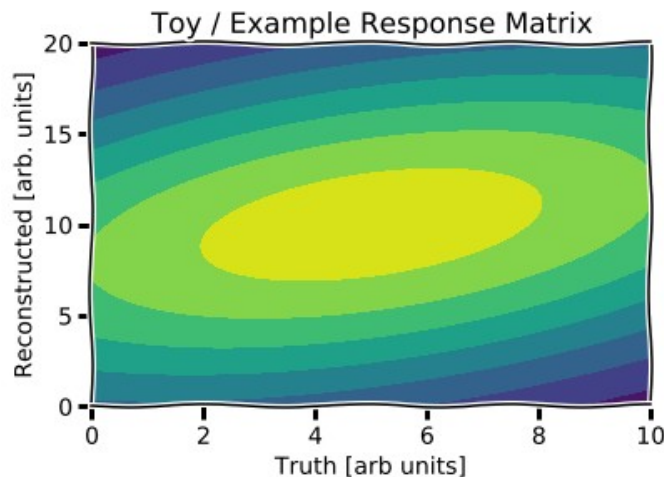
- Both forward- and unfolding depend on the detector response matrix
- In this analysis we build the response matrix by comparing MC truth to reconstructed MC



- Unfolding provides not model parameters, but physical quantities
- Unfolding constrains data via parameters
 - Unlike forward folding we cannot fit values of parameters
- Based on our systematic uncertainties, we constrain our unfolded bin content

- Bayesian Unfolding (D'Agostini)

$$P_0(i|j) = \frac{P(j|i) \times P_0(i)}{P_0(j)}$$

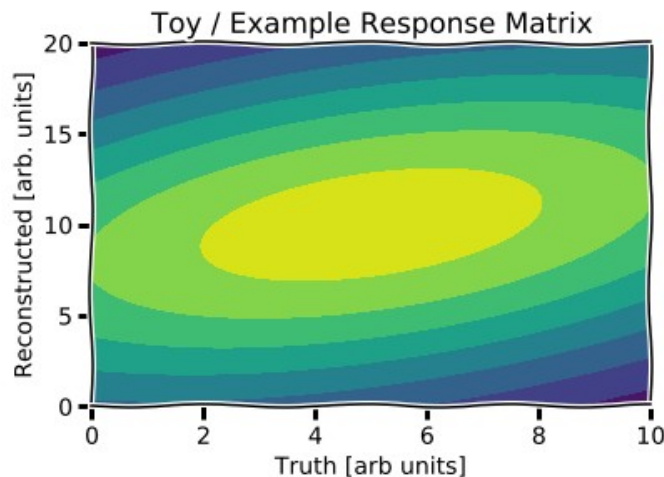


- Prior information about our measurement: If we draw a sample from the MC truth distribution, what is the probability to be drawn from bin i ?
- Normalization: Probability to observe an event in bin j . Given by the logic of requiring an event in true bin i **and** for that event to contribute to bin j , and summing for all bins. (This is why it's fair to call it a normalization constant)
- All terms known to precision of MC

i = truth bin
 j = reco. bin

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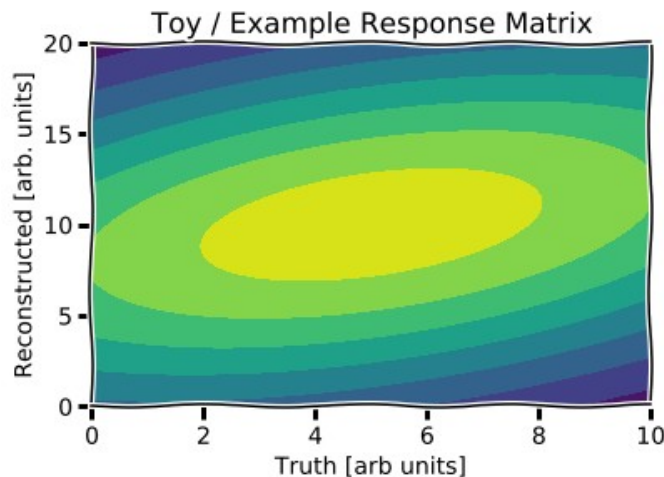
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$$P_0(i) = \frac{N_i^{MC}}{\sum_k N_k^{MC}}$$



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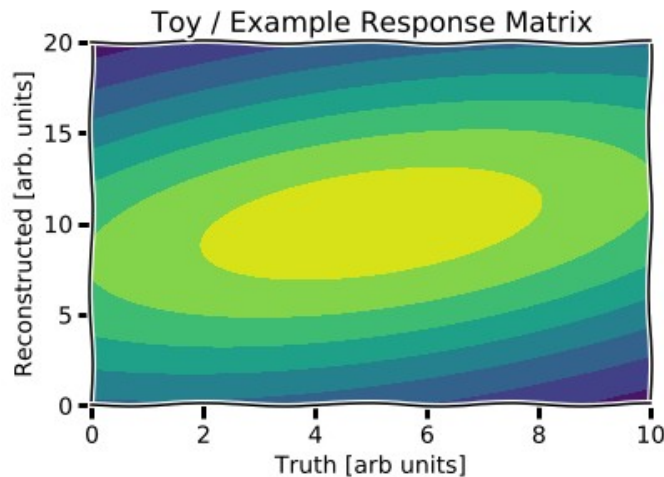
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- Bayesian Unfolding (D'Agostini)

$$P_0(i|j) = \frac{P(j|i) \times P_0(i)}{P_0(j)}$$



$$P_0(j) = \sum_i [P(j|i) \times P(i)]$$



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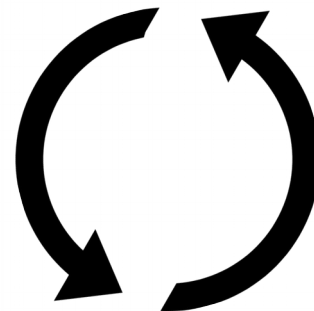
- Generates unfolding matrix via Bayes' theorem

$$P_0(i|j) = \frac{P(j|i) \times P_0(i)}{P_0(j)}$$

- Estimates unfolded spectrum U, from measurement M:

$$U_n = P_n(i|j) \times M$$

- Output of each step is prior for next step.
- Biased towards MC for low iterations → Bias drops with iterations
- Statistical Uncertainty → Grows with number of iterations
- Final number of iterations must be a trade-off between the above two



- Response matrix constructed from MC

- Reco-side: reconstructed $\cos(\theta_z)$, reconstructed energy and reconstructed track length
- Truth-side: MC PID, MC truth $\cos(\theta_z)$ and MC truth energy

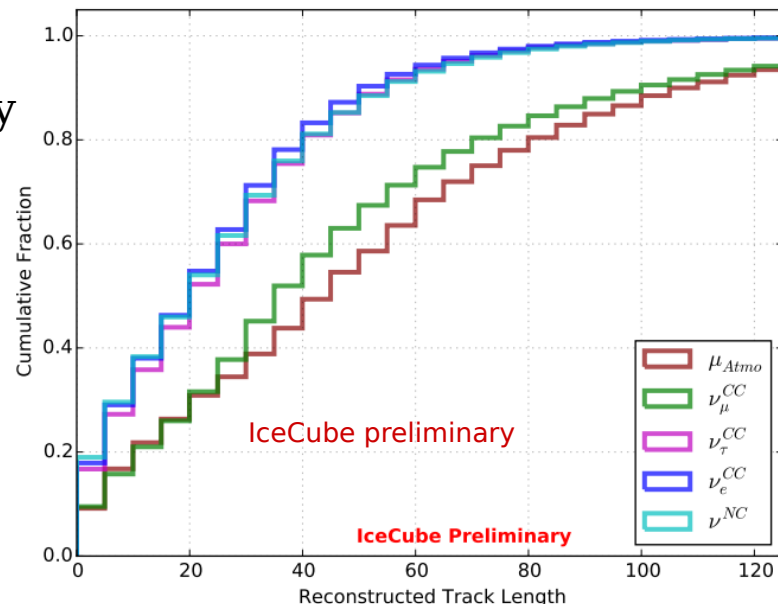
- PID channels:

$$\nu_{\mu}^{CC} + \bar{\nu}_{\mu}^{CC} \quad \text{and} \quad \nu_{rest} + \bar{\nu}_{rest}$$

- Weight truth side by: $(t_{live} V_{IC})^{-1}$

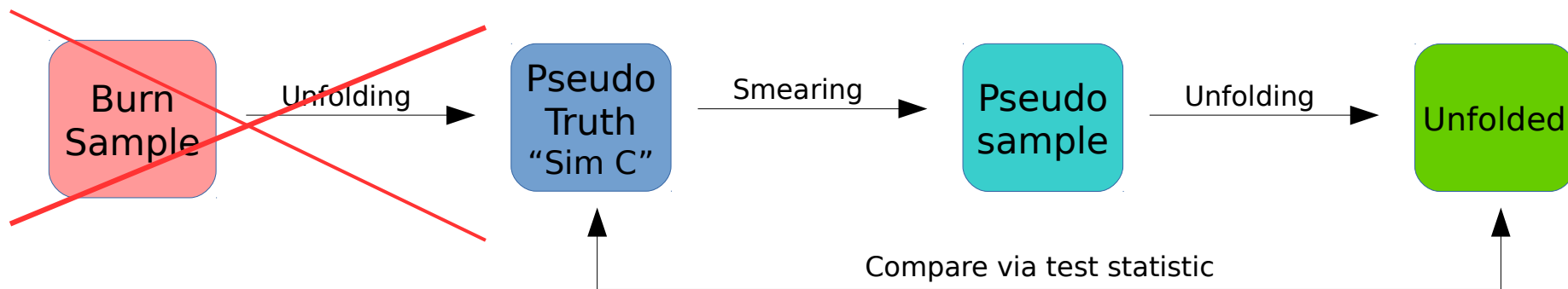
- Unfolded quantity is “True In-ice interaction rate per volume [m^3/s]”

- From these distributions the energy and zenith spectra are calculated



TSU: A Blind Burn Sample Closure Test

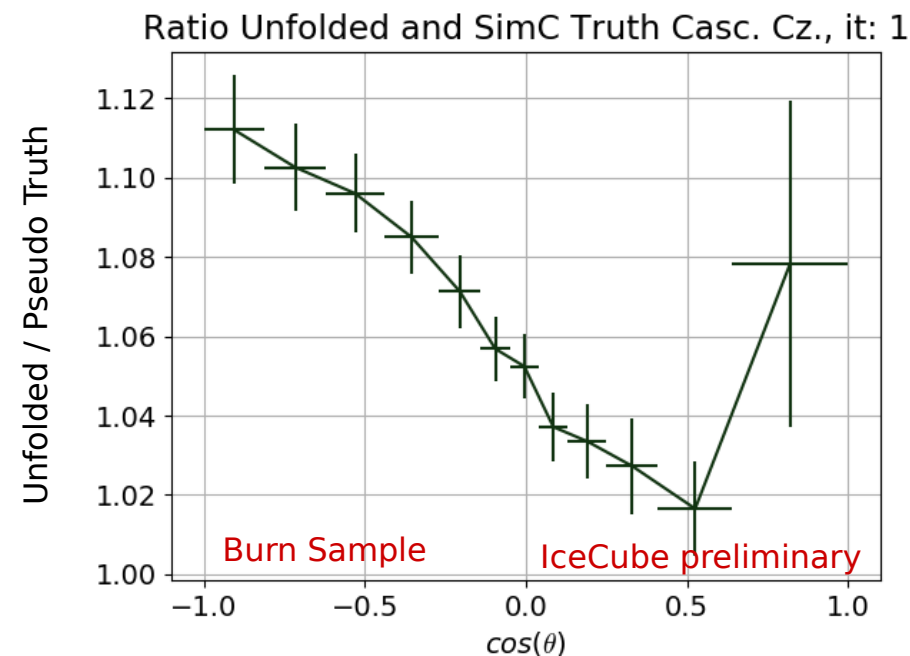
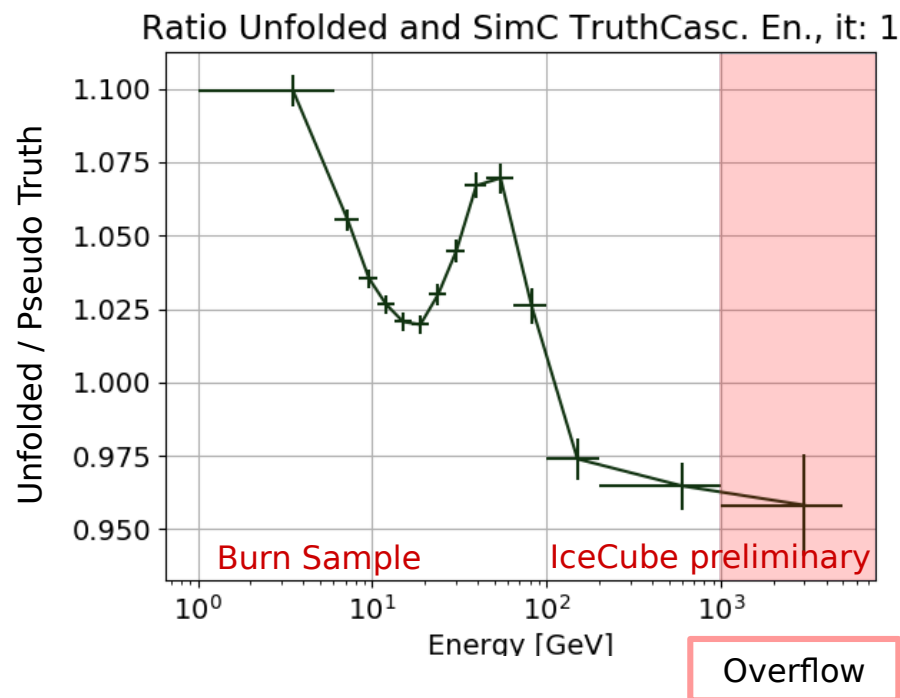
- Problem: When unfolding a real data sample we do not have access to truth information like in the MC case
- Aim: Show stability of unfolding method across smearing and unfolding
- Closure test: Truth-Smeared-Unfolded (TSU) test



- Unfold to 25 iterations
- Result: Converges on pseudo-truth to well within statistical uncertainty
- Careful consideration of stopping condition is necessary

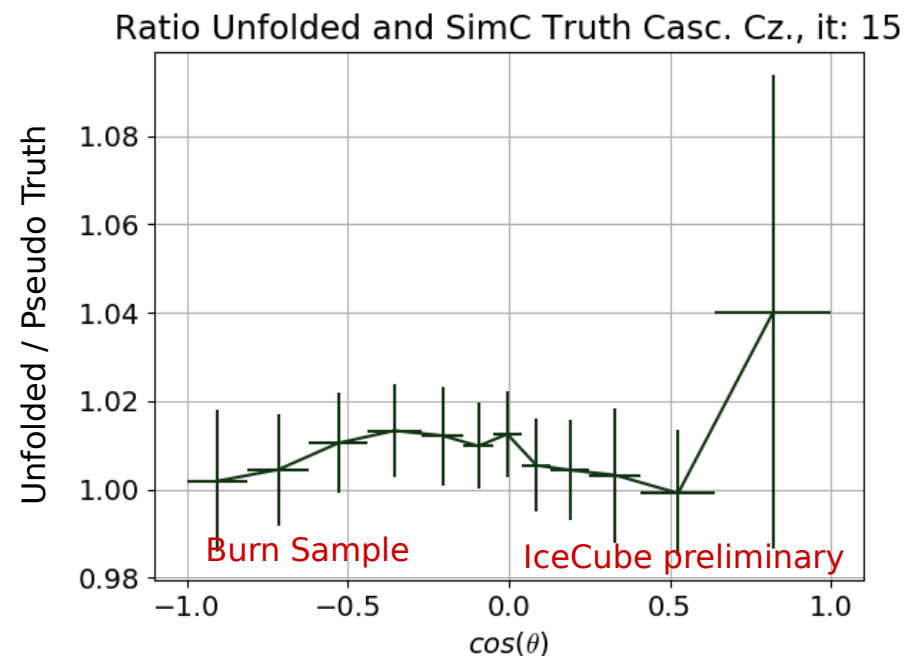
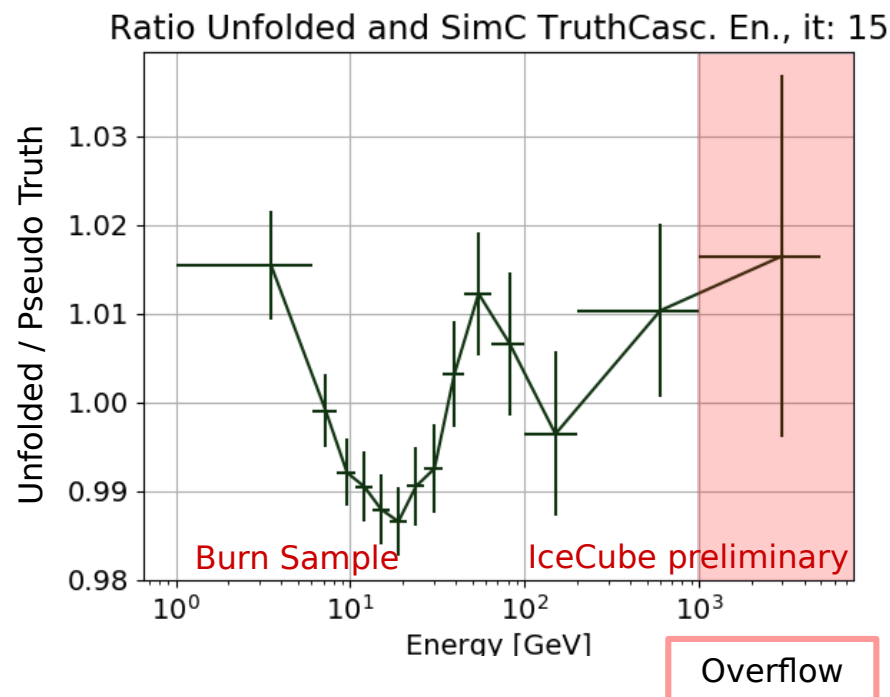
TSU: Burn Sample Consistency

- Blind Check: TSU-Ratios, 1 iteration,
- Checks consistency in unfolding



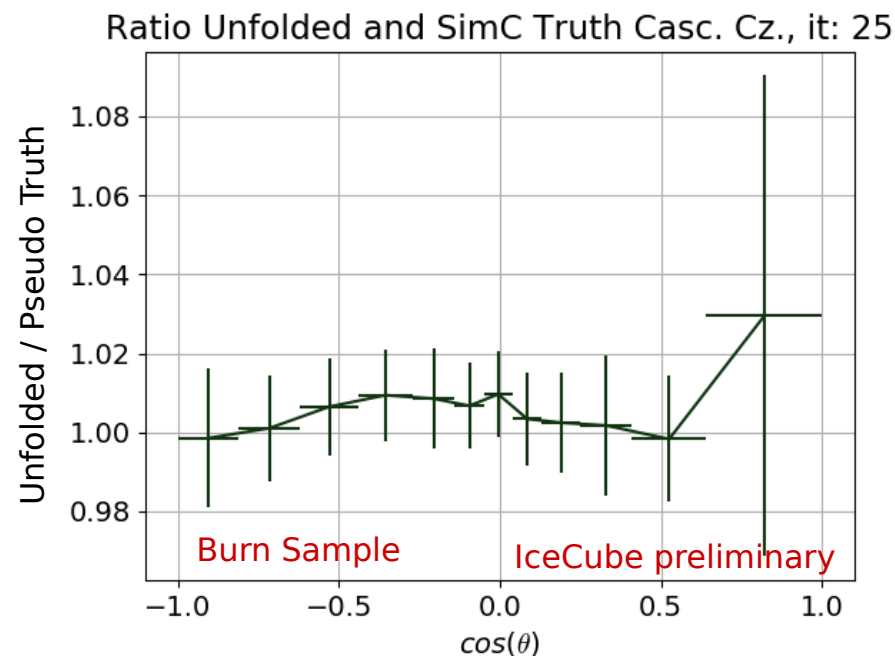
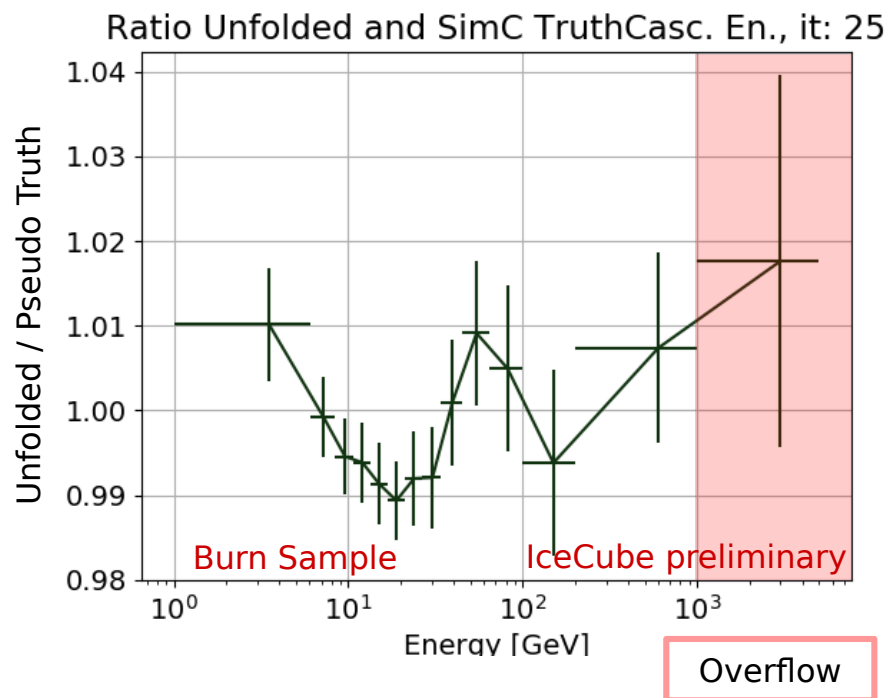
TSU: Burn Sample Consistency

- Blind Check: TSU-Ratios, 15 iterations,
- Reasonable consistency



TSU: Burn Sample Consistency

- Blind Check: TSU-Ratios, 25 iterations,
- Reasonable Consistency



Setting the Stopping condition

- Two test statistics:

- Statistics only:

$$TS = \frac{1}{UM} \sum_i \frac{(m_i U - u_i M)^2}{m_i + u_i}$$

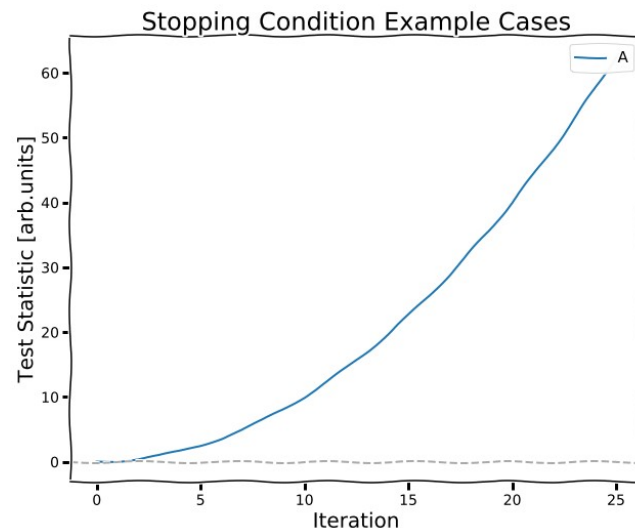
- Uncertainty based:

$$TS_{\sigma} = \frac{1}{N} \sum_i \left(\frac{\sigma_i^{max} - \sigma_i^{min}}{n_i} \right)^2$$

- The stopping condition plots consist of calculating a test statistic for every iteration between unfolded and pseudo truth - can take many different shapes.

Hierarchy of procedure:

- In case of divergence: Stop after 4 iterations, as advised by D'Agostini (A)



Setting the Stopping condition

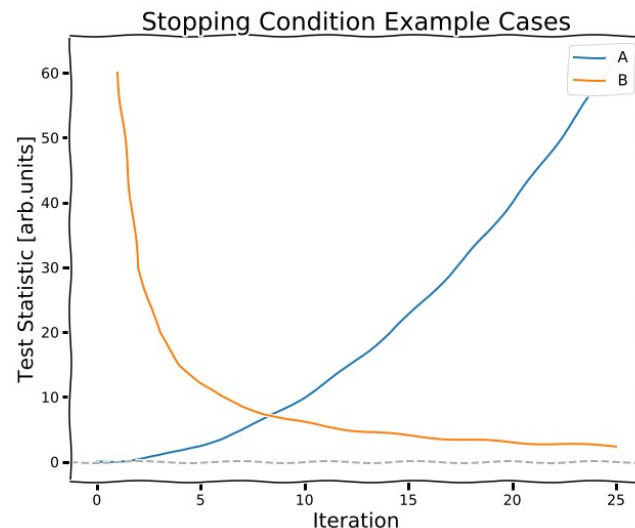
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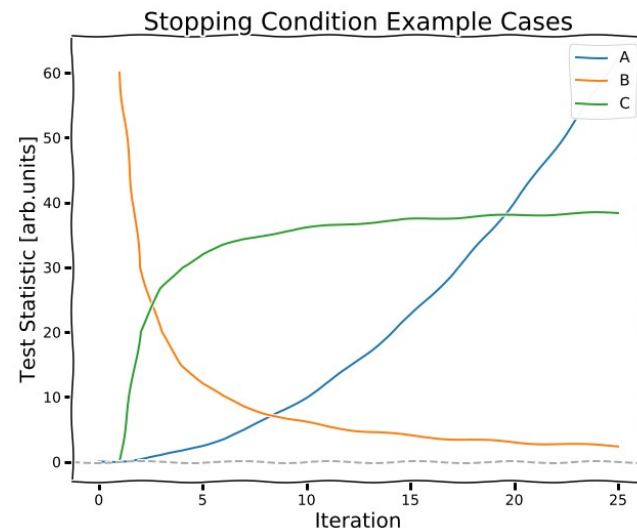
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- In case of convergence: Use systematics based stopping condition. (B)
- In case of systematic dominance after 1 iteration: Use statistics only stopping condition. (C)



Setting the Stopping condition

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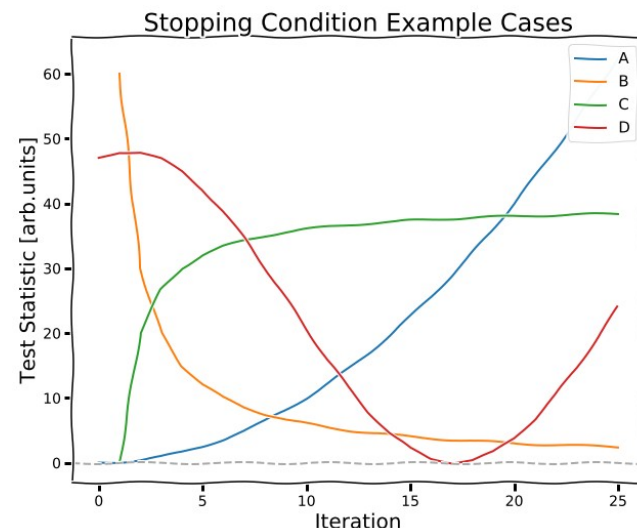
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Hierarchy of procedure:

- In case of divergence: Stop after 4 iterations, as advised by D'Agostini (A)
- In case of convergence: Use systematics based stopping condition. (Minimize distribution) (B)
- In case of systematic dominance after 1 iteration: Use statistics only stopping condition (C)
- Otherwise: Minimize the distribution (D)



Setting the Stopping condition

- Two test statistics:

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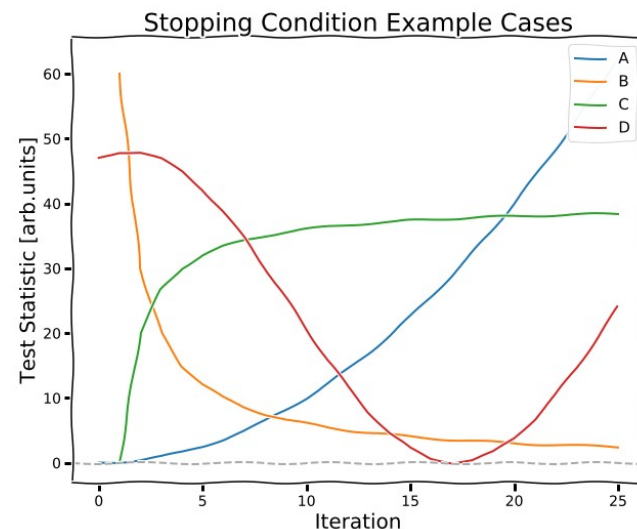
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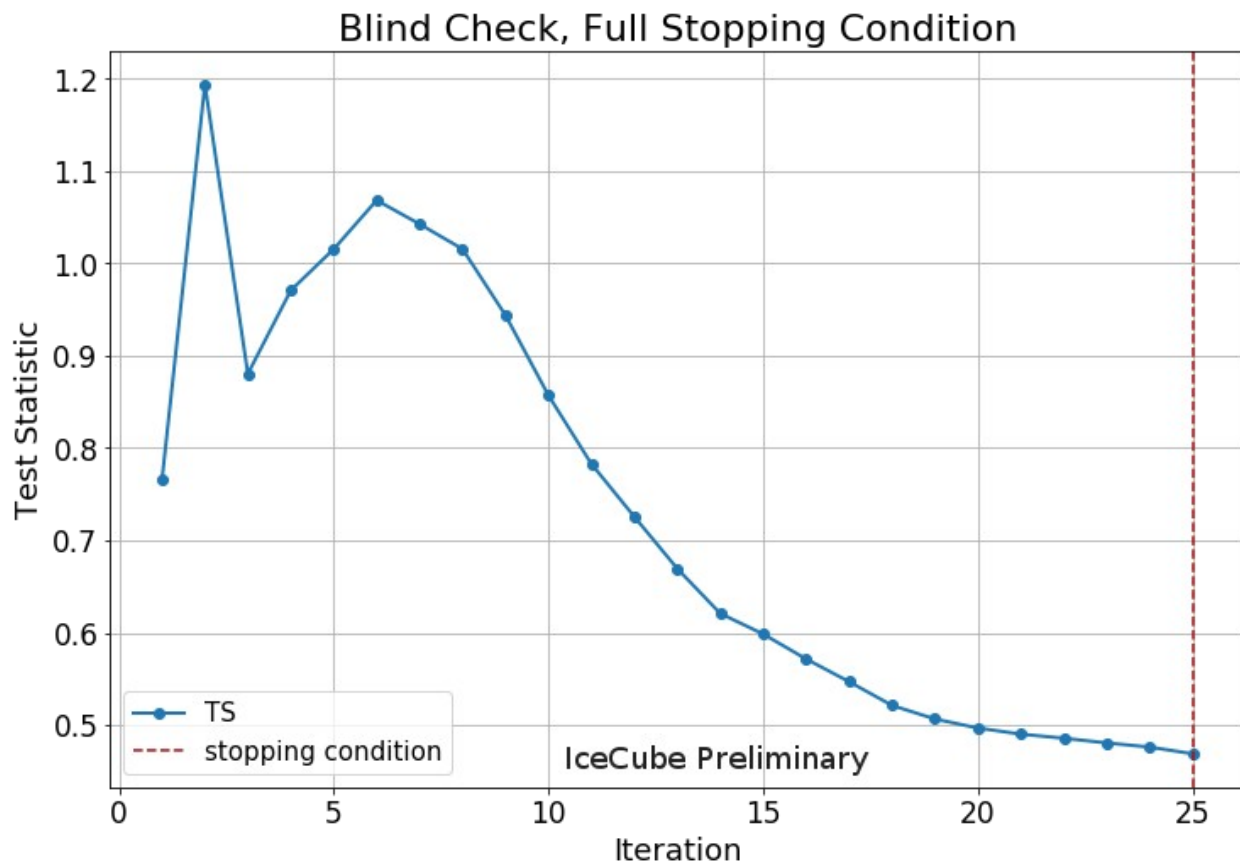
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- Otherwise: Minimize the distribution (D)
- Burn sample test case: D, minimum at 20 iterations



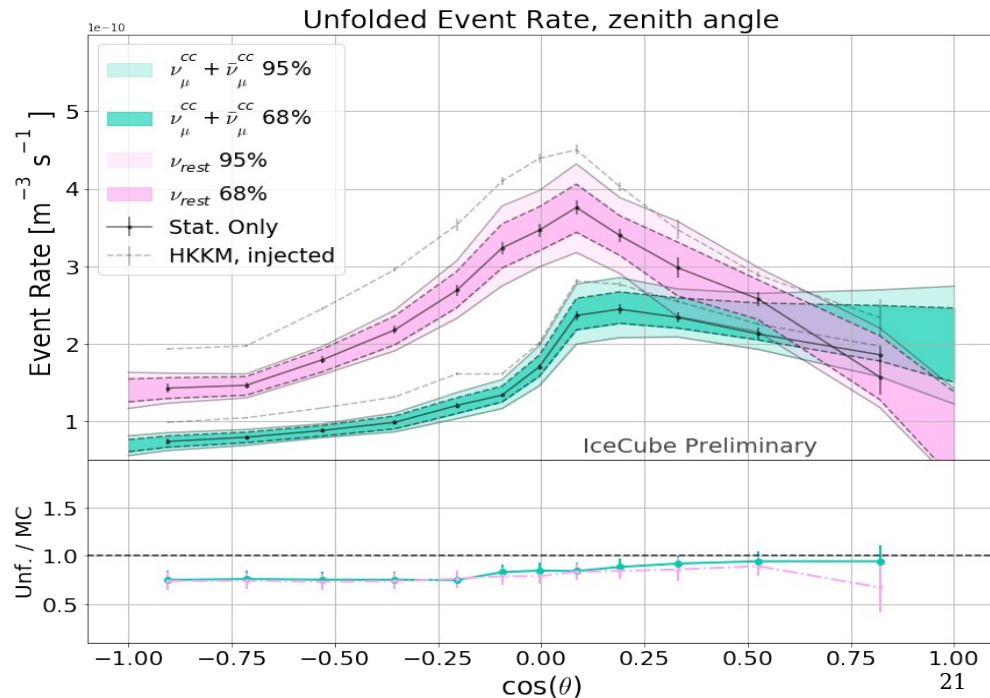
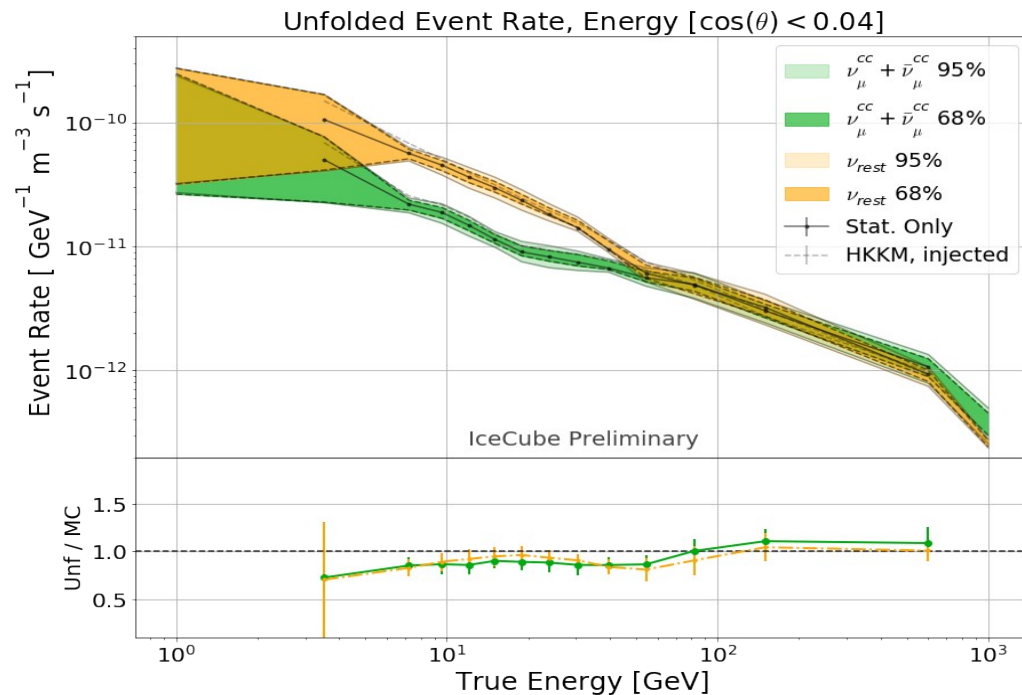
Stopping condition

- For full data sample
- Only small variation above ~ 20 iterations.



Unfolded Event Rate

- 2 channels based on idealized event signature in detector
- $\nu_{\mu}^{cc} + \bar{\nu}_{\mu}^{cc}$
- Everything else: ν_{rest}
- ~1400 trials

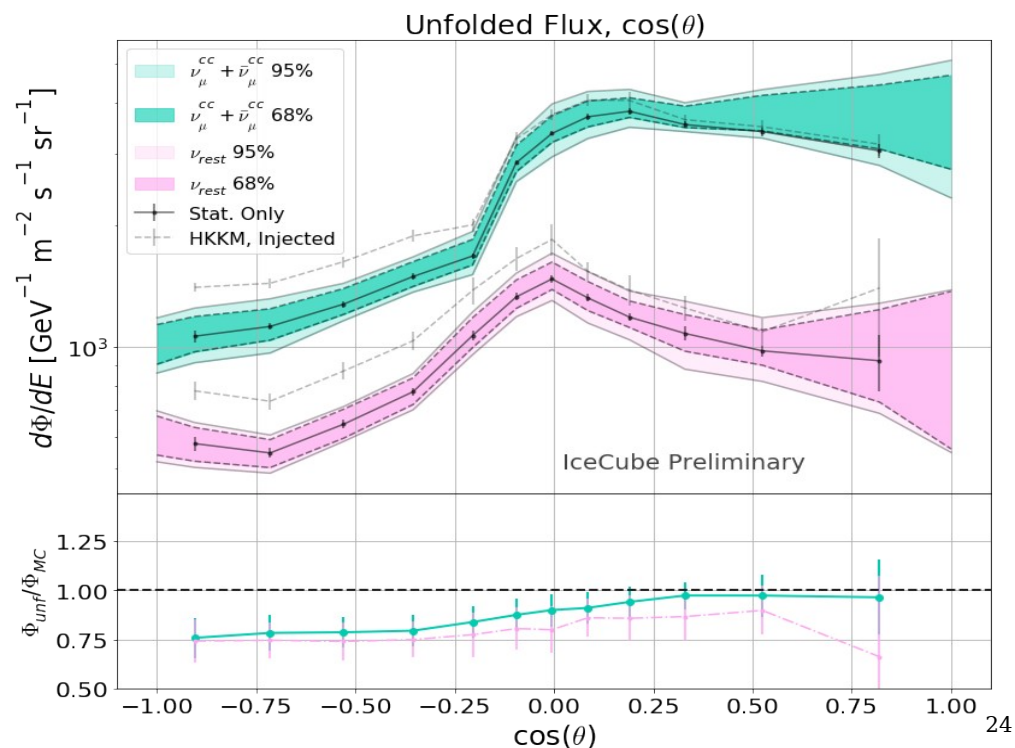
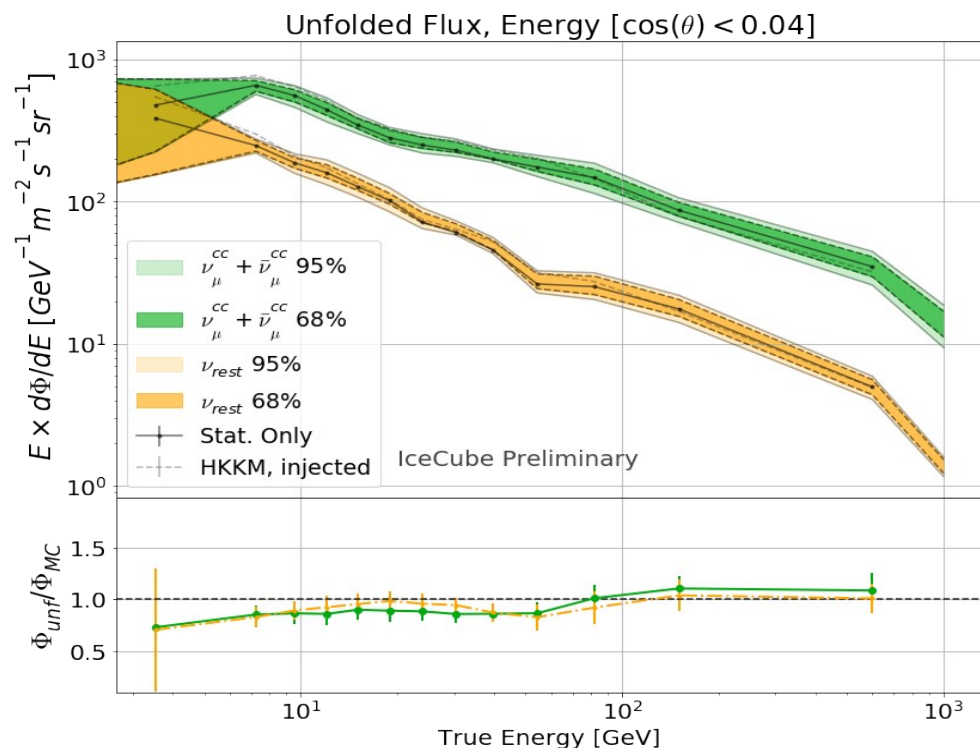


- Unfolded measurement of atmospheric neutrino flux at south pole
- Allows model builders to test predictions on many parameters
- Some tension with expectation below 10 GeV and in up going region
- Data release and publication in preparation

- From here

Addendum: Unfolded Flux

- Same 2 channels, now also compensated for cross sections
- $\nu_{\mu}^{cc} + \bar{\nu}_{\mu}^{cc}$
- Everything else: ν_{rest}
- ~ 75 trials



- Random sampling of systematics due to non-linear effects
- ~1400 trials

Oscillation and Weighting Systematics		
Systematic	Value	Prior
θ_{12}	34.5°	$\pm 1.1^\circ$ ⁽¹⁾
θ_{23}	41°	$\pm 0.11^\circ$ ⁽¹⁾
θ_{13}	8.41°	$\pm 0.17^\circ$ ⁽¹⁾
Δm_{21}^2	$7.56 \text{ e}^{-5} \text{ eV}^2$	$\pm 0.19 \text{ e}^{-5} \text{ eV}^2$ ⁽¹⁾
Δm_{31}^2	$2.55 \text{ e}^{-3} \text{ eV}^2$	$\pm 0.04 \text{ e}^{-3} \text{ eV}^2$ ⁽¹⁾
δ_{cp}	252°	$\pm 24^\circ$ ⁽¹⁾
Livetime	4.8 [yr]	1%
Muon Scale	1.0	5%
Noise	1.0	10%

Discrete Systematics		
Systematic	Value	Prior
Dom eff	1.0	10%
Hole ice	25	± 5
Bulk ice scattering	1.0	10%
Bulk ice absorption	1.0	10%

Analysis Chain:

Sample



Osc. re-weighting



Discrete sys.



Unfolding

- Definiton:

$$\epsilon = \frac{N_{sel}}{N_{gen}} = \frac{R_{sel} V_{fidu}}{R_{gen} V_{gen}}$$

- With the number of efficiency corrected unfolded events:

$$N_{unf}^{\epsilon} = \frac{N_{unf}}{\epsilon} = \frac{R_{unf} V_{fidu}}{\left(\frac{R_{sel} V_{fidu}}{R_{gen} V_{gen}} \right)} = \frac{R_{unf} R_{gen} V_{gen}}{R_{sel}}$$

- To get a rate independent from the volume, it is prudent to modify the efficiency; we divide out the generator volume to arrive at:

$$\epsilon' = \frac{N_{sel}}{N_{gen}/V_{gen}} = \epsilon V_{gen} \longrightarrow R_{unf}^{\epsilon'} = \frac{N_{unf}}{\epsilon V_{gen}} = \frac{R_{unf} R_{gen}}{R_{sel}}$$