
Three Dimensional Global Test Particle Simulation of Cosmic-Ray Acceleration and Escape in Supernova Remnants

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Acceleration and Escape in SNRs

Cosmic Ray Acceleration in SNRs

Main acceleration region

Parallel shock

High injection rate?

Slow acceleration?

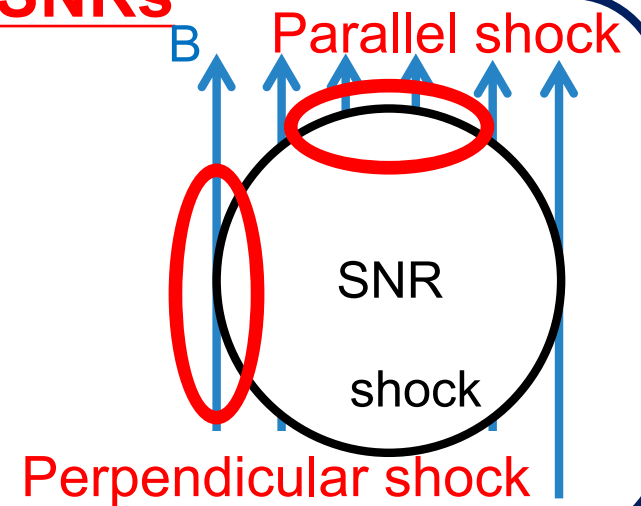
vs.

We focus on

Perpendicular shock

Low injection rate?

Rapid acceleration!!!



Cosmic Ray Escape from SNRs

Escape is important to determine the CR spectra and the maximum energy.

Escape from the perpendicular shock has never been investigated.

In this study, we investigate

- 1) the acceleration time at a perpendicular shock.
- 2) the escape from the perpendicular shock region.

Energy Spectrum of a Perp. Shock Acceleration

The energy spectra of a perpendicular shock acceleration becomes **sorter than that of the standard DSA prediction** in the case that **the magnetic fluctuation is weak in downstream region.**

ref: Takamoto & Kirk (2015)

Observations and simulations about a downstream region suggest that **the magnetic field is amplified and is stronger than that of a upstream region.**

ref: observation : Bamba et al.(2003), Ohira and Yamazaki(2017)

simulation : Ohira (2016), Caprioli and Spitkovski(2013), Inoue et al.(2009)
Giacalone and Jokipii(2007)



In this study, we assume

**the strong magnetic field amplification in a downstream region and
the random walk in the downstream region.**

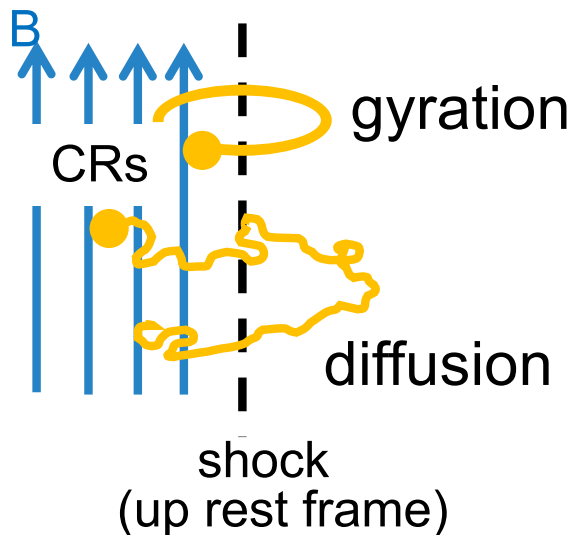
Acceleration Time

Acceleration mechanism : DSA $\frac{\Delta p}{p} \sim \frac{u_{sh}}{c}$

Assumption : The residence time in downstream region is negligible.

(because the magnetic field is amplified in the downstream region.)

Acceleration time $t_{acc} = \frac{p}{\Delta p / \Delta t}$ Δt : the residence time in upstream region



	residence time in up region	acceleration time
gyration	$\frac{2\pi}{\Omega_g}$	$\frac{2\pi c}{\Omega_g u_{sh}}$ ref: Ohira(2016)
diffusion	$\frac{4D_{\perp}}{u_{sh} c}$	$\frac{4D_{\perp}}{u_{sh}^2}$ ref: Drury (1983)

Maximum Energy at a Perpendicular Shock

	gyration	diffusion
acceleration time t_{acc}	$\frac{2\pi c}{\Omega_g u_{\text{sh}}}$	$\frac{4D_{\perp}}{u_{\text{sh}}^2}$

$$t_{\text{acc}} \sim t_{\text{age}}$$



t_{age} : SNR age

$$D_{\perp} = \xi D_{\parallel} \quad \xi : \text{const.}$$

$$D_{\parallel} = \frac{\eta_B}{3} r_g c \quad \eta_B : \text{Bohm factor}$$

ref: Giacalone & Jokipii(1999)

gyration $E_{\text{max}} \sim 10^{15} \text{eV} \left(\frac{B_0}{3\mu\text{G}} \right) \left(\frac{u_{\text{sh}}}{0.01c} \right) \left(\frac{t}{1\text{kyr}} \right)$

diffusion

For Kolmogorov with $\delta B/B_0=1$,

$$\left[\xi \sim 0.02, \quad \eta_B \sim \left(\frac{B_0^2}{\delta B^2} \right)_{\text{res}} \sim \left(\frac{r_g}{L_c} \right)^{-2/3} \right] \quad L_c : \text{the injection scale of turbulence}$$

$$E_{\text{max}} \sim 10^{13} \text{eV} \left(\frac{\xi}{0.02} \right)^{-3} \left(\frac{B_0}{3\mu\text{G}} \right) \left(\frac{L_c}{10\text{pc}} \right)^{-2} \left(\frac{u_{\text{sh}}}{0.01c} \right)^6 \left(\frac{t}{1\text{kyr}} \right)^3$$

Simulation Setup for a Plane Shock Case

- forward shock velocity

$$u_{sh} = 0.01c \text{ (constant)}$$

- time resolution

$$\Delta t = 0.01 \Omega_{g0}^{-1} \text{ (all region)}$$

assumption

Bohm diffusion for $100B_0$ in down region

$$\text{scattering time : } t_{scat} = \Delta t(p/p_0)$$

- impulsive injection @t=0

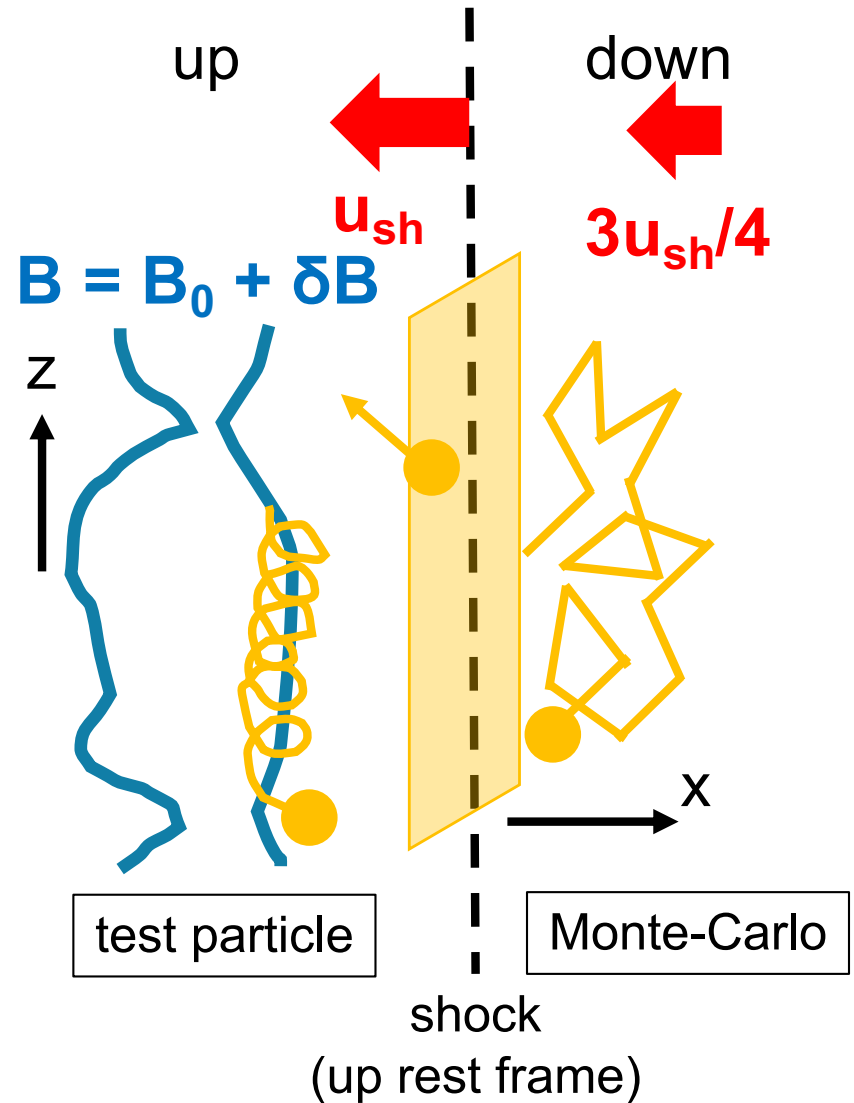
$$\gamma_0 = 1.4, \text{ isotropic}$$

- Magnetic fluctuation

isotropic Kolmogorov spectrum

injection scale of turbulence $L_c : 0.1pc$

$$\begin{aligned} \sigma &= (\sum_n \delta B(k_n)^2) / B_0^2 \\ &= \delta B_{tot}^2 / B_0^2 \\ &= 0.01, 1 \end{aligned}$$



Magnetic Field

➤ back ground: $\vec{B}_0 = (0, 0, B_0)$ B_0 :const.

➤ turbulence: summation of static transverse waves

ref : Husein & Shalchi (2014)

$$\delta\vec{B}(x, y, z) = \text{Re} \sum_{n=1}^{N_m} \delta B(k_n) \vec{\xi}_n \exp[i(k_n z'_n + \beta_n)]$$

(

- N_m :the number of mode
- $\vec{\xi}_n$:polarization vector
- β_n : phase($0 \sim 2\pi$)

)

 (

- k_n :wave number
- $A(k_n)$:amplitude

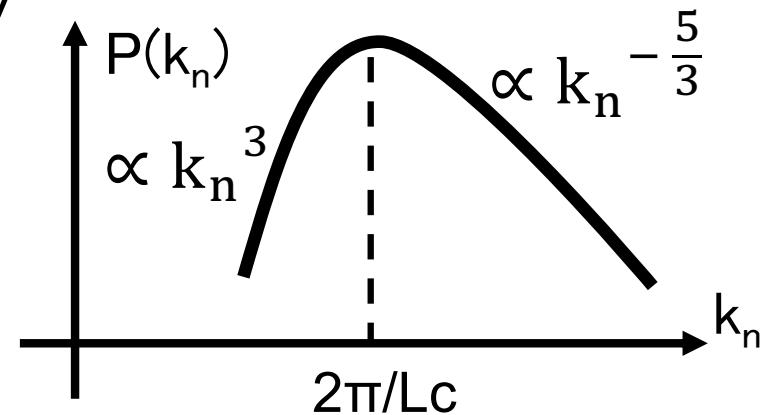
)

▪ spectrum $G(k_n)$: isotropic Kolmogorov

$$\delta B(k_n)^2 = \sigma B_0^2 \frac{G(k_n) \Delta k_n}{\sum_{i=1}^{N_m} G(k_i) \Delta k_i}$$

$$G(k_n) = \frac{(k L_c / 2\pi)^3}{[1 + (k L_c / 2\pi)^2]^{7/3}}$$

$$\sigma = (\sum_n \delta B(k_n)^2) / B_0^2$$

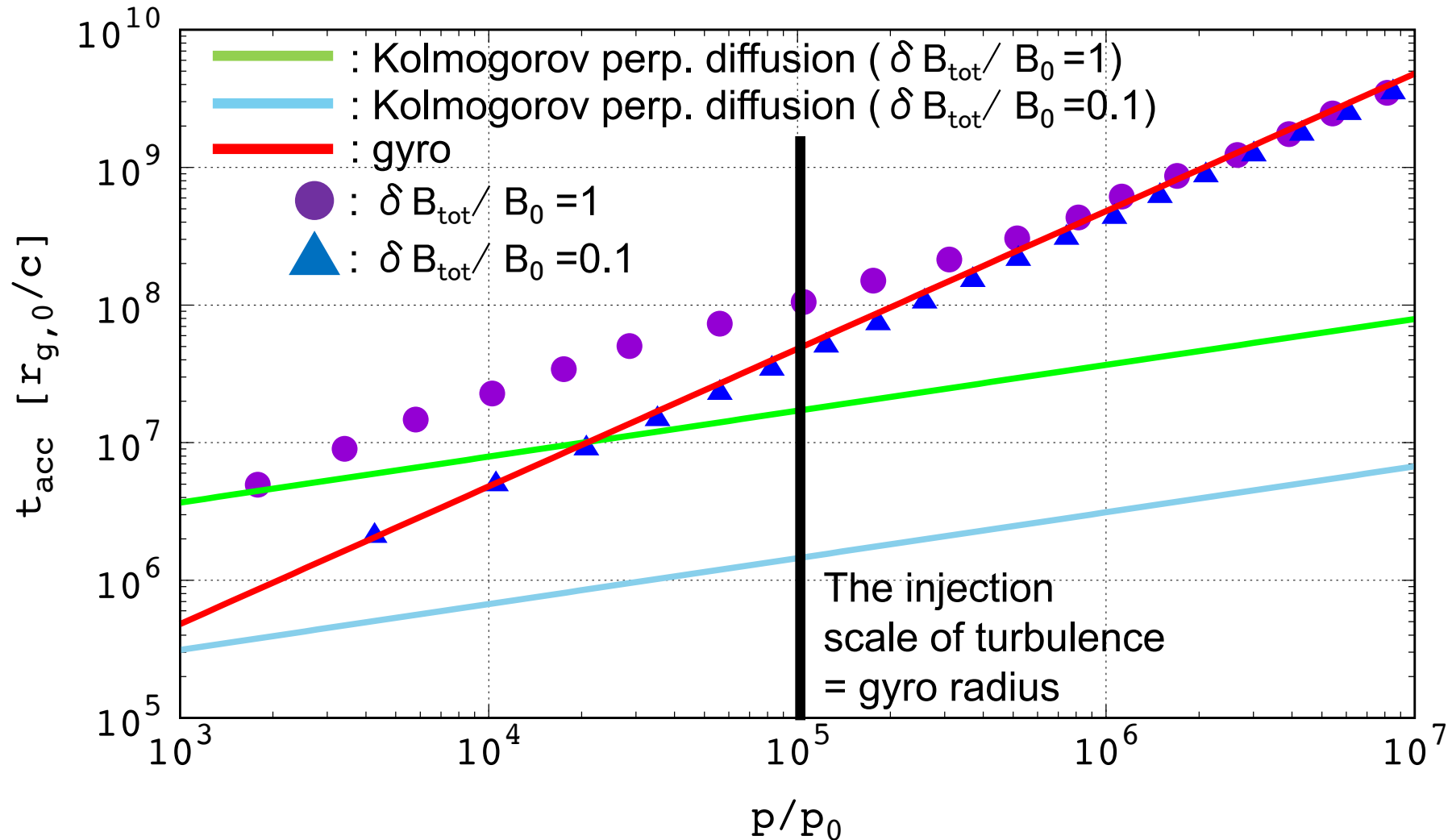


L_c : the injection scale
of turbulence

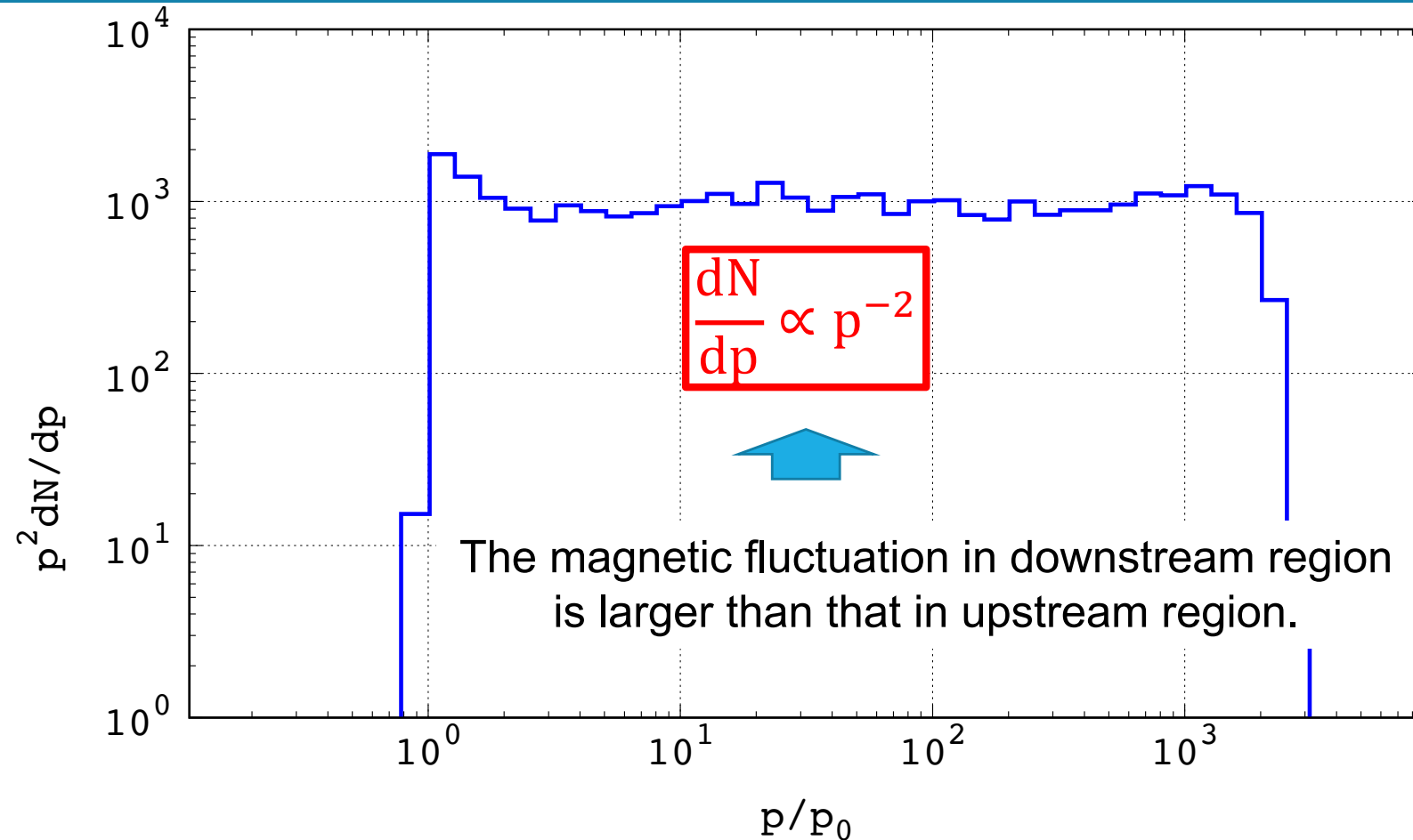
Simulation Result

In weak fluctuation case,

- 1) The diffusion approximation is NOT valid.
- 2) More efficient acceleration occurs.
- 3) Particles are gyrating in the upstream region to get energy.



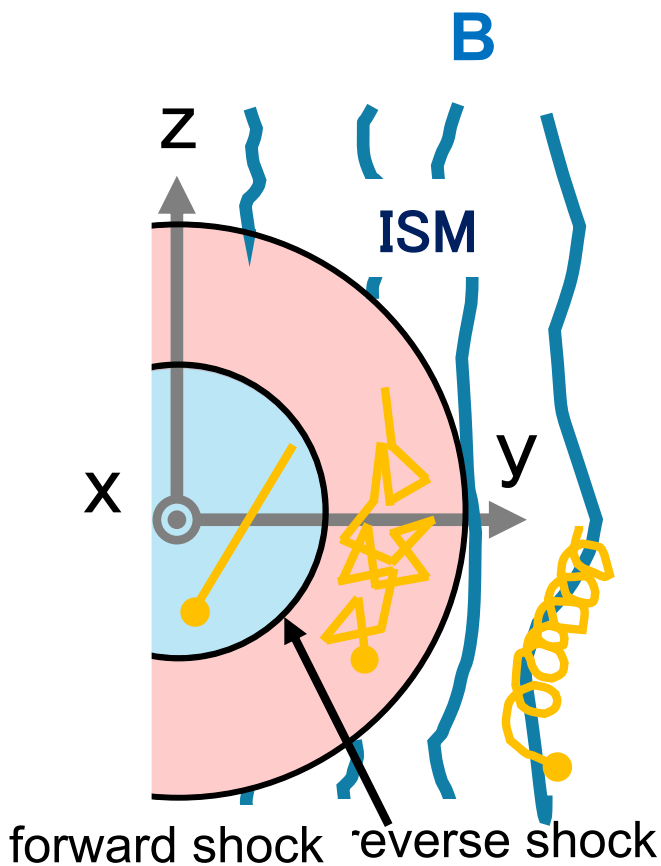
Spectrum of accelerated particles



The rapid acceleration is compatible with the canonical spectrum, p^{-2} , by the strong magnetic fluctuation in downstream region.

It does not depend on the strength of the upstream magnetic fluctuation.

Global Simulation for the Perpendicular Shock Acc. in an SNR



The velocity field is given by a simple model.
(ref : Ohira et al. (2018)).

ISM region : test particle simulation

E.O.M
$$\frac{d\vec{u}}{dt} = \frac{q}{m \gamma c} \vec{u} \times \vec{B}$$

Freely expanding ejecta region

The reverse shock has not pass through this region.

- ➡ The magnetic field is very weak.
- ➡ The free streaming is assumed.

Shocked region

The magnetic field is amplified to $100B_0$



Isotropic scattering in the downstream rest frame.
→ Monte Carlo method

$t_{\text{scat}}(p) = 0.01 \Omega_{g0}^{-1} (p/p_0)$ (The Bohm limit is assumed.)

Simulation Setup ($E_{\text{SN}}=10^{51}$ erg, $M_{\text{ej}}=1M_{\text{sun}}$, $n_0=1\text{cm}^{-3}$)

impulsive injection

(The forward shock surface of the equatorial plane at the initial time)

Initial velocity : $\gamma_0=149$ (140GeV), isotropic

time resolution : $0.01 \Omega_{g0}^{-1}$ (all region)

Ω_{g0} : gyro frequency of the injected particle

magnetic fluctuation

wave number :

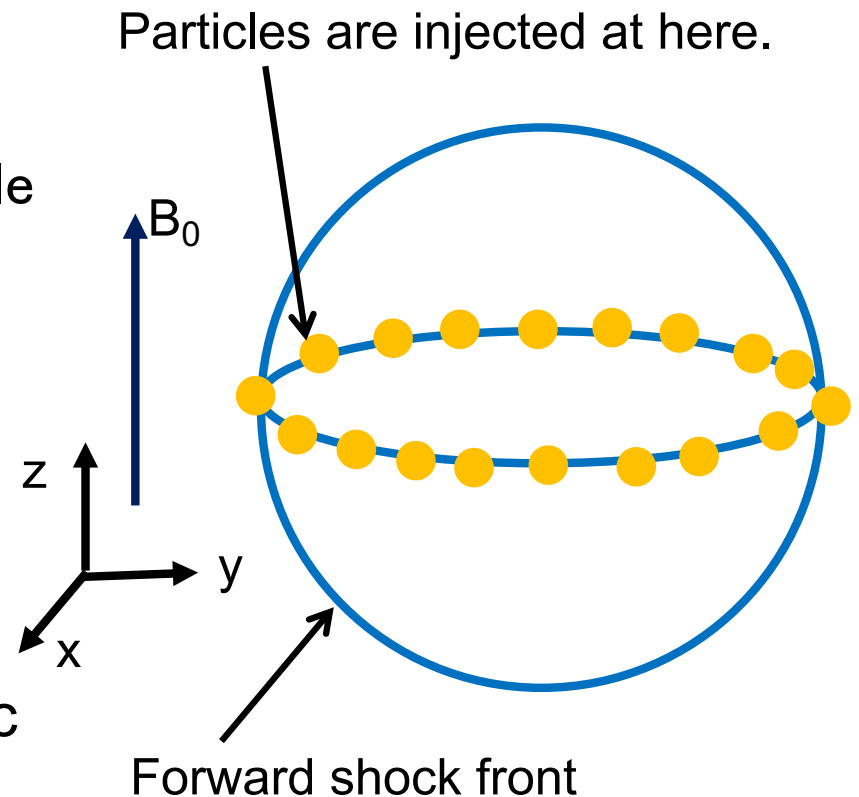
$$2\pi \times 10^{-5}/r_{g0} < k < 2\pi \times 10/r_{g0}$$

spectrum : isotropic Kolmogorov

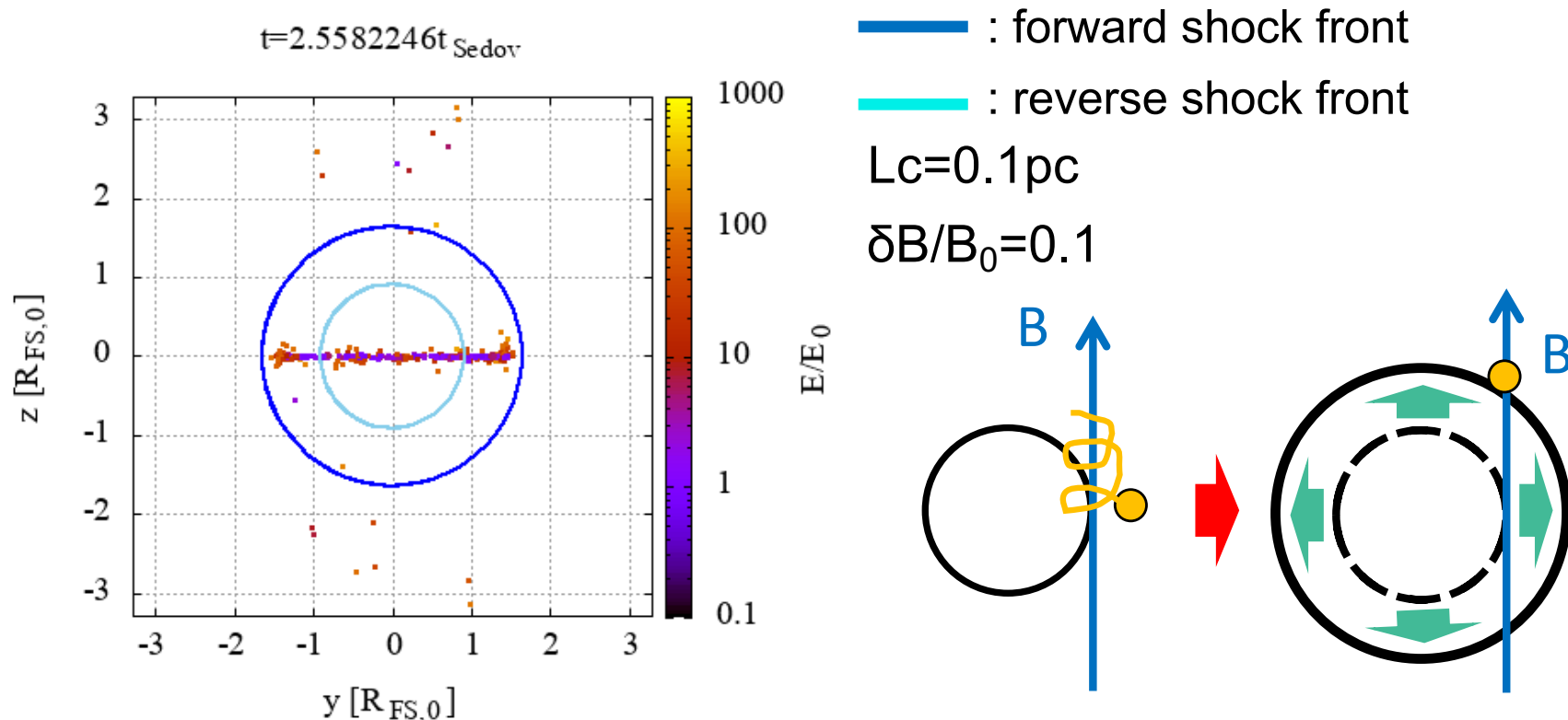
injection scale of turbulence L_c : 0.1pc

the number of modes : 240

$$\sigma = \delta B_{\text{tot}}^2 / B_0^2 = 0.01$$



Evolution of Particle Position ($t/t_{\text{Sedov}}=1-3$)



1, The injected particles move along the uniform magnetic field.

→ escape from the perpendicular shock region.

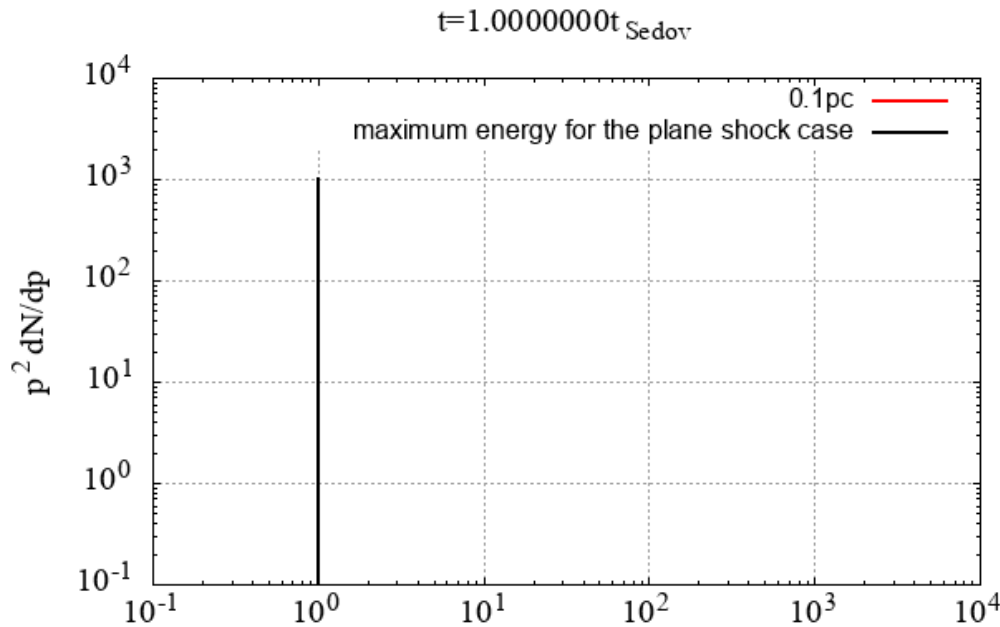


2, shock expands.



3, The escaping particles seem to be escaping from the parallel shock region.

Evolution of Energy Spectrum ($t/t_{\text{Sedov}}=1-3$)



The rapid acceleration at a perp. shock region stops in a short time.

Maximum Energy

$$E_{\text{max}} \sim 200E_0 \sim 30 \text{ TeV}$$

($E_0=140\text{GeV}$)

$$t_{\text{acc}} = t_{\text{esc}} \Leftrightarrow \frac{2\pi c}{\Omega_g u_{\text{sh}}} = \frac{R_{\text{SNR}}^2}{4D_{\parallel}} \quad (D_{\parallel} \sim r_g c) \quad R_{\text{SNR}} : \text{SNR radius}$$

$$E_{\text{max}} \sim 50 \text{ TeV} \left(\frac{B}{1\mu\text{G}} \right) \left(\frac{R_{\text{SNR}}}{2\text{pc}} \right) \left(\frac{u_{\text{sh}}}{0.02c} \right)^{1/2}$$

In this case, the acceleration at the perpendicular shock is limited by the diffusion along the magnetic field line.

Summary and Future Work

Summary

- **The diffusion approximation is not valid** for a perp. shock acceleration in the case that the magnetic fluctuation is weak in the upstream region.
- The efficient acceleration occurs in the case that **the magnetic field fluctuation is weak in the upstream region**
- The energy spectrum at perp. shock become **E⁻²** in the case that **the magnetic fluctuation is strong in downstream region.**
- The rapid acceleration occurs at the perp. shock region of an SNR, the accelerated particles escape from the perp. shock region along the magnetic field line. $t_{\text{esc,perp}} \sim R_{\text{SNR}}^2 / (4D_{\parallel})$

Future Work

- We are going to perform more simulations for other parameter cases. (e.g. injection scale, injection time and strength of the magnetic fluctuation)
- We are going to investigate the time evolution of E_{max} at the perp. shock and energy spectra of escaping CRs from the perpendicular shock region.